Fachbereich Mathematik
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11. Exercise sheet Analysis II for MCS
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(G11.1)

Compute the arc length of the following paths.

(i) $f : [0, 2\pi] \to \mathbb{R}^3$, $f(t) = (r\cos t, r\sin t, ct)$, where $r, c > 0$.

(ii) $g : [0, 2\pi] \to \mathbb{R}^2$, $g(t) = (t - \sin t, 1 - \cos t)$.

Solution.

(i) Since f is continuously differentiable, we use Theorem 8.21. We have $f'(t) = (-r\sin t, r\cos t, c)$ and $\|f'(t)\|_2^2 = r^2 + c^2$. Hence

$$L(f) = \int_0^{2\pi} \sqrt{r^2 + c^2}\mathrm{d}t = 2\pi\sqrt{r^2 + c^2}.$$

(ii) Again we can use Theorem 8.21, since g is continuously differentiable. We have $g'(t) = (1 - \cos t, \sin t)$ and $\|g'(t)\|_2^2 = 2 - 2\cos t = 4\sin^2 \frac{t}{2}$. Hence

$$L(g) = \int_0^{2\pi} \sqrt{4\sin^2 \frac{t}{2}}\mathrm{d}t = 2\int_0^{2\pi} \sin\frac{t}{2}\mathrm{d}t = 4[-\cos\frac{t}{2}]_0^{2\pi} = 8.$$

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(G11.2)

Let (X, d) be a metric space, and let $a \le b \in \mathbb{R}$ and $c \le d \in \mathbb{R}$. Let $\gamma' : [c, d] \to X$ be a path obtained from a path $\gamma : [a, b] \to X$ by a change of parameter. Prove that $L(\gamma) = L(\gamma')$.

(This is Proposition 8.13 in the handouts.)

Solution. Handwritten.

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(G11.3)

Let (X, d) be a metric space, and let $a \le b \in \mathbb{R}$. Let $\gamma : [a, b] \to X$ be a path in X. Prove that for all $c \in [a, b]$ we have

$$L(\gamma) = L(\gamma|_{[a,c]}) + L(\gamma|_{[c,b]}).$$

(Recall that $\gamma|_{[a,c]} : [a, c] \to X$ is the restriction of γ to $[a, c]$, i.e. $\gamma|_{[a,c]}(x) = \gamma(x)$ for $x \in [a, c]$.)

(This is Proposition 8.17 in the handouts.)

Solution. Handwritten.

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