Fachbereich Mathematik
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7. Exercise sheet Analysis II for MCS
Summer Term 2006

(G7.1)

Compute the following integrals.

(i)

$$\int_0^{1/2} \arcsin x \, \mathrm{d}x.$$

(ii)

$$\int_0^1 \frac{6x^2+4}{x^3+2x+1} \, \mathrm{d}x.$$

(iii)

$$\int_a^b \frac{1}{1-x^2} \, \mathrm{d}x,$$

where $a < b \in \mathbb{R}$ and $-1, 1 \notin [a,b]$.

Solution. Handwritten.

■

(G7.2)

Let $a < b \in \mathbb{R}$ and let $I_m := \int_a^b \sin^m x \, \mathrm{d}x$ for $m \in \mathbb{N}_0$. Give a formula which expresses I_m for $m > 1$ in terms of I_n for $n < m$.

Solution. Handwritten.

■

(G7.3)

Let $a \in]0, \infty[$ and let F be an antiderivative of the continuous function $f : [-a, a] \to \mathbb{R}$ with $F(0) = 0$. Prove

(i) If f is odd, i.e. $f(x) = -f(-x)$ for $x \in [-a, a]$, then F is even, i.e. $F(x) = F(-x)$ for $x \in [-a, a]$.

(ii) If f is even, then F is odd.

Solution. From Theorem 4.41 and the Fundamental Theorem we conclude that F is given by $F(x) = \int_0^x f(t)\mathrm{d}t$.

(i) We compute by using the substitution $s = -t$.

$$F(-x) = \int_0^{-x} f(t)\mathrm{d}t = \int_0^x -f(-s)\mathrm{d}s = \int_0^x f(s)\mathrm{d}s = F(x).$$

Therefore, F is even.

(ii) By the same substitution as in (i) we get

$$F(-x) = \int_0^{-x} f(t)\mathrm{d}t = \int_0^x -f(-s)\mathrm{d}s = -\int_0^x f(s)\mathrm{d}s = -F(x),$$

which means that F is odd. ■