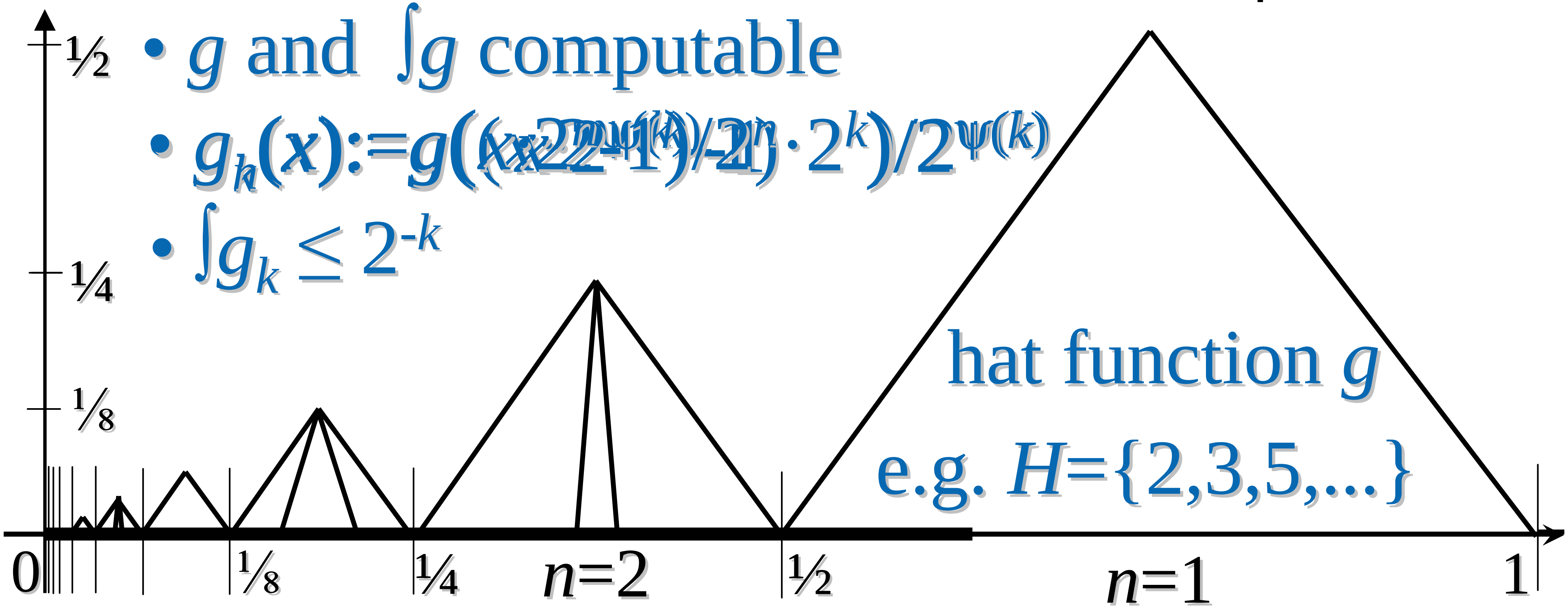
Myhill'71: uncomputable ∂ on $C^1[0,1]$

TECHNISCHE UNIVERSITÄT DARMSTADT
Martin Ziegler

Fact : ∃ computable bijection $\psi:\mathbb{N}\to H$

- g and $\int g$ computable
- $g_n(x):=g((x-2^{-n})\cdot 2^n)$ / $g_k(x):=g((x\cdot 2^k)/2^{\psi(k)})$
- $\int g_k \leq 2^{-k}$



$h':=\sum_{n\in H} g_n$ / $h':=\sum_k g_k$ continuous, incomputable,

yet $h:=\int h' \in C^1[0;1]$ computable.

q.e.d.

The Case of the Wave Equation

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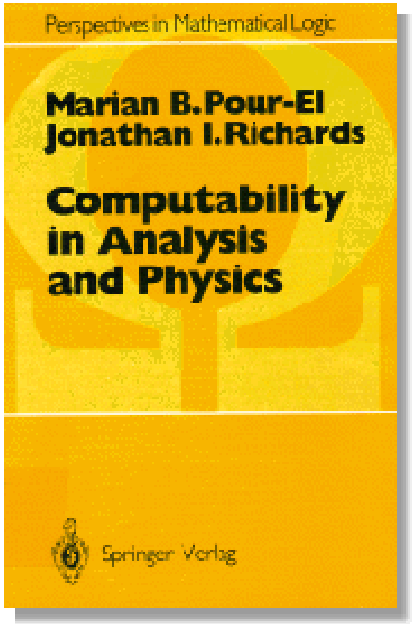
Martin Ziegler

Myhill'71: computable $h \in C^1[0,1]$ with uncomputable $h'(1)$

Pour-El&Richards'81 construct a computable $f \in C^1(\mathbb{R}^3)$ such that for g:=0 the unique solution is *in*computable at t=1 and $\underline{x}$=(0,0,0).



<u>Church-Turing Hypothesis</u> (Kleene):
Everything that can be computed by a Turing machine can also be computed by a physical device – and vice versa!

$$\partial^2/\partial t^2\, u(\underline{x},t) = \Delta u(\underline{x},t), \quad u(\underline{x},0)=f(\underline{x}), \quad \partial/\partial t\, u(\underline{x},0)=g(\underline{x})$$

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Kirchhoff's formula:

$$u(t,\vec{x}) = \frac{\partial}{\partial t}\left(\frac{1}{4\pi t}\int_{|\vec{y}-\vec{x}|=t} f(\vec{y})\,d\sigma(\vec{y})\right) + \frac{1}{4\pi t}\int_{|\vec{y}-\vec{x}|=t} g(\vec{y})\,d\sigma(\vec{y})$$

$$\boxed{f(\vec{x}) := h(|\vec{x}|^2)}$$

$$u(t,0) = \frac{d}{dt}\left(h(t^2)\cdot t\right) = h'(t^2)\cdot 2t^2 + h(t^2)$$

$\partial^2/\partial t^2\, u(\underline{x},t) = \Delta u(\underline{x},t)$, $u(\underline{x},0)=f(\underline{x})$, $\partial/\partial t\, u(\underline{x},0)=g(\underline{x})$

Two Effects in Real Computability

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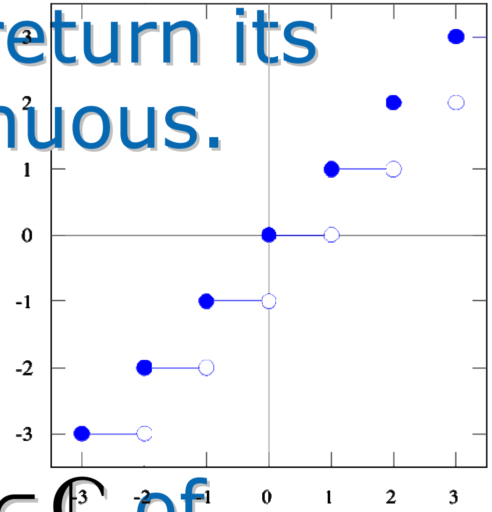
a) Multivalued 'functions'

Example floor function: given $x \in \mathbb{R}$, return its least integer upper bound — discontinuous.



Given x, return <u>some</u> integer upper bound: computable!

Example fund. theorem of algebra: Given $a_0,\ldots a_{d-1} \in \mathbb{C}$, return roots $x_1,\ldots x_d \in \mathbb{C}$ of $a_0 + a_1 \cdot X + \ldots + a_{d-1} \cdot X^{d-1} + X^d \in \mathbb{C}[X]$ incl. multiplicities

up to permutation [Specker'67]

b) Discrete 'advice'

Example matrix diagonalization: given $A \in \mathbb{R}^{d\cdot(d-1)/2}$, return a basis of eigenvectors — discontinuous:

$$\varepsilon \cdot \begin{pmatrix} \cos(1/\varepsilon) & \sin(1/\varepsilon) \\ \sin(1/\varepsilon) & -\cos(1/\varepsilon) \end{pmatrix}$$

Thm: Computable knowing $|\sigma(A)|$.