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Computable Analysis

SS 2013, Exercise Sheet #8

EXERCISE 16:

a) Let $\tilde{\xi} :\subseteq \mathrm{LM} \to X$ and $\tilde{\upsilon} :\subseteq \mathrm{LM} \to Y$ be second-order representations.
Define the product second-order representation $\tilde{\xi} \times \tilde{\upsilon}$ of $X \times Y$.

b) Prove that both addition $+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and multiplication are second-order polytime $(\tilde{\rho} \times \tilde{\rho}, \tilde{\rho})$–computable. How about exponentiation?

c) For ordinary representations ξ of X and υ of Y consider the induced second-order representations $\tilde{\xi}$ and $\tilde{\upsilon}$ according to the lecture.
For an arbitrary but fixed $f :\subseteq X \rightrightarrows Y$ prove that polytime (ξ, υ)–computability is equivalent to second-order polytime $(\tilde{\xi}, \tilde{\upsilon})$–computability.

d) Verify that second-order polynomials are closed under *both* kinds of composition

$$\big(Q \circ P\big)(n,\lambda) \;:=\; Q\big(P(n,\lambda),\lambda\big) \quad \text{and} \quad \big(Q \bullet P\big)(n,\lambda) \;:=\; Q\big(n, P(\cdot,\lambda)\big) \;. \tag{1}$$

e) Explore the asymptotic growth of $n \mapsto P(n,\lambda)$ for linearly bounded $\lambda : n \mapsto n + c$ in terms of n and c. How about $\lambda : n \mapsto a \cdot n + c$? Prove that, whenever λ is a polynomial (of degree d, say) then so is $n \mapsto P(n,\lambda)$. How does its degree vary with d?