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Computable Analysis

SS 2013, Exercise Sheet #6

EXERCISE 14:

a) Prove that the composition of polynomial-time computable total $f, g : \{\mathtt{0}, \mathtt{1}\}^* \to \{\mathtt{0}, \mathtt{1}\}^*$ is again polynomial-time computable.

b) Compare the computational complexity of the Hamiltonian Cycle problem HC with that of the Hamiltonian Path problem:

$$\text{HP} \;=\; \big\{\langle G\rangle : G \text{ graph admits a path visiting every vertex precisely once.}\big\}$$

c) Construct a polynomial-time reduction from EC (Eulerian Cycle) to HC.

d) A graph with *integer edge weights* is a mapping $E : V \times V \to \mathbb{N}$. The *weighted length* of a path $(v_1, \ldots, v_m)$ is $\sum_{j=1}^{m-1} E(v_j, v_{j+1})$.
Construct a polynomial-time reduction from HC (Hamilton Cycle) to the following *Traveling Salesperson Problem*:

$$\text{TSP} \;=\; \big\{\langle \text{bin}(E), \text{bin}(k)\rangle : E \text{ weighted graph admits a Hamiltonian cycle of length} \leq k\big\}$$

e) A graph (V, E) is k-colorable if it admits a mapping $c : V \to \{1, \ldots, k\}$ with $c(u) \neq c(v)$ for all $(u, v) \in E$. Formalize the problem of whether a given graph is 3-colorable and construct a polynomial-time reduction to SAT.