

## Computable Analysis

### SS 2013, Exercise Sheet #5

#### EXERCISE 13:

- a) Let  $\|\cdot\|$  in Equation (4) of the script denote any fixed computable norm.  
 Let  $\|\cdot\|'$  denote some other norm on  $\mathbb{R}^d$  with induced representation  $\psi_{>}^d$ .  
 Then i)  $\psi_{>}^d \preceq \psi_{>}^d$  and ii)  $\psi_{<}^d \preceq \psi_{<}^d$ .
- b) Prove that union  $\mathcal{A}^{(d)} \times \mathcal{A}^{(d)} \ni (A, B) \mapsto A \cup B \in \mathcal{A}^{(d)}$  is  $(\psi_{<}^d \times \psi_{<}^d, \psi_{<}^d)$ -computable.
- c) How about intersection (under what reasonable restrictions)?
- d) *Not* using Theorem 4.7j) from the script,  
 prove that  $f^{-1}[0]$  is  $\psi_{<}^d$ -computable for every computable  $f: \mathbb{R}^d \rightarrow \mathbb{R}$ .
- e) Prove that  $\{\emptyset\}$  is  $\psi_{>}^d|_{[0;1]^d}$ -r.e.  
 Hint: Cmp. the proof to Exercise 2...