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Computable Analysis

SS 2013, Exercise Sheet #4

EXERCISE 11:
Fix representations α of X and β of Y and γ of Z.

a) Prove that type conversion

$$R_{\alpha\times\beta,\gamma}[X\times Y,Z]\times X \ \ni\ (g,x)\ \mapsto \big(Y\ni y\mapsto g(x,y)\in Z\big)\ \in\ R_{\beta,\gamma}[Y,Z]$$

is well-defined and $\big([\alpha\times\beta\to\gamma]\times\alpha,[\beta\to\gamma]\big)$–computable.

b) Also the converse conversion

$$R_{\alpha,[\beta\to\gamma]}\big[X,R_{\beta,\gamma}[Y,Z]\big]\ \ni\ \big(X\ni x\mapsto g(x,\cdot)\in R_{\beta,\gamma}[Y,Z]\big)$$
$$\mapsto\ \big(X\times Y\ni(x,y)\mapsto g(x,y)\in Z\big)\ \in\ R_{\alpha\times\beta,\gamma}[X\times Y,Z]$$

is well-defined and $\big([\alpha\to[\beta\to\gamma]],[\alpha\times\beta\to\gamma]\big)$–computable.

EXERCISE 12:

a) Prove that the (discontinuous) Heaviside Function $h:\mathbb{R}\to[0;1]$ is $(\rho_<,\rho_<)$–computable, where

$$h(x)=0 \text{ for } x\leq 0 \quad\text{ and }\quad h(x)=1 \text{ for } x>0\ .$$

b) Prove that every $(\rho,\rho_<)$–computable $f:[0;1]\to\mathbb{R}$ is lower semi-continuous.

c) Prove that every $(\rho_<,\rho)$–computable $f:[0;1]\to\mathbb{R}$ is constant.

d) Prove $\rho_<\not\preceq\rho_>$ using a discontinuity/adversary argument.