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Computable Analysis

SS 2013, Exercise Sheet #1

EXERCISE 1:
Prove the following (nonuniform) effective intermediate value theorem:
A computable function $f : [0;1] \to [-1;1]$ with $f(0) \cdot f(1) < 0$ has a computable root.

EXERCISE 2:
Let $(r_m)_m$ and $(\varepsilon_m)_m$ be computable rational sequences such that

$$[0;1] \setminus \bigcup_m (r_m - \varepsilon_m, r_m + \varepsilon_m) \quad = \quad \{x_0\} \ .$$

Conclude that this x_0 is a computable real number.

EXERCISE 3:
Prove that every computable function $f : [0;1] \to \mathbb{R}$ with finitely (or countably) many roots has a computable root.

EXERCISE 4:
Let $\mathbb{R}_c$ denote the set of computable reals. Prove that $\mathbb{R}_c$ is a *real closed* field.