Algorithmic Discrete Mathematics
7. Exercise Sheet

TECHNISCHE UNIVERSITÄT DARMSTADT

Department of Mathematics
Andreas Paffenholz
Silke Horn

SS 2013
10/11 July 2013

Groupwork

Exercise G1

You are given twelve coins, eleven of which have identical weight, one is lighter or heavier. How many weighings with a binary scale do you need to find the deviating coin (and the sign of the deviation)? Draw the decision tree and prove that your number of weighings is optimal.

Extra puzzle: Find a solution without case distinctions.

Exercise G2

Let $n \geq 1, q \geq 2, \ell_1, \dots, \ell_n \in \mathbb{Z}_{\geq 0}$. Prove:

(a) If $T \in T(n,q)$ has leaves of lengths $\ell_1, \dots, \ell_n$ then $\sum_{i=1}^{n} q^{-\ell_i} \leq 1$. Equality occurs if and only if T is complete.

(b) Given lengths $\ell_1, \dots, \ell_n$ such that $\sum_{i=1}^{n} q^{-\ell_i} \leq 1$, there is $T \in T(n,q)$ with these lengths.

Exercise G3

Assume you want to encode the symbols $a, \dots, z$ over the alphabet $A = \{0, 1, 2\}$.

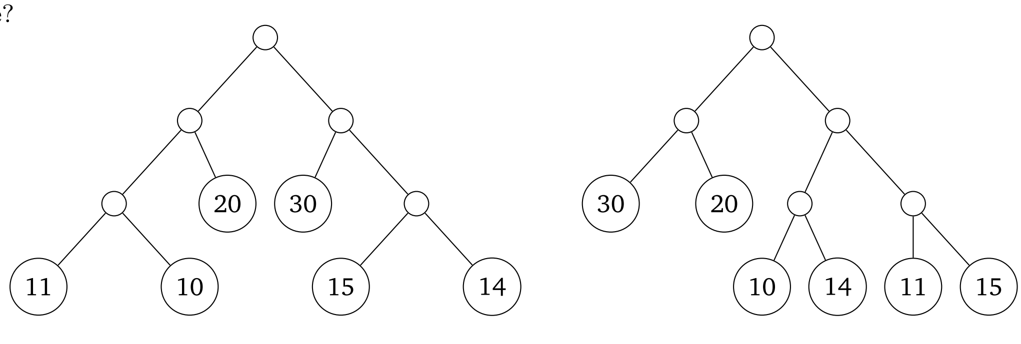
(a) Devise an equal length code (*i.e.,* every symbol is encoded with a code word of equal length) for this problem and encode the following sentence using your code (without punctuation and case sensitivity):

The lecturer's name is Andreas Paffenholz.

(b) Now construct an optimal prefix code for the above sentence using Huffman's Algorithm by counting the occurrences of the letters, encode the sentence again and compare the length of the output with the equal length code above.

Exercise G4

Given $p_1, \dots, p_6 = 10, 11, 14, 15, 20, 30$ show that the following decision trees are both optimal. Which one is also a Huffman tree?



Exercise G5

Consider the following game for two players: Start with a natural number $n > 1$. Each player takes turns to divide the current number by a power of a prime number (different from 1). If the current number is 1 at the beginning of one player's round, then that player loses.

(a) Draw the game tree for $n = 12$. What is the best strategy for the first player?

(b) Play the number game using different values of n. Can you find a game (*i.e.,* an n) for which the second player always wins?