Algorithmic Discrete Mathematics
1. Exercise Sheet

TECHNISCHE UNIVERSITÄT DARMSTADT

Department of Mathematics
Andreas Paffenholz
Silke Horn

SS 2013
17/18 April 2013

Groupwork

Exercise G1
Show that a graph $G = (V, E)$ is bipartite if and only if it does not contain cycles of odd length.

Exercise G2
Let $G = (V, E)$ be a graph. Prove:

(a) Any walk with distinct endpoints v, w contains a path between v and w.

(b) Any closed walk contains a cycle.

Exercise G3
A walk in a connected graph $G = (V, E)$ is called an *Eulerian trail* if it contains each edge of G exactly once. A closed Eulerian trail is called an *Eulerian tour*. The graph G is called *Eulerian* if it contains an *Eulerian tour*.

(a) Which of the graphs in Figure 1 are Eulerian.

(b) Let G be a connected graph. State conditions for G to be Eulerian and prove that these conditions are necessary.

(c) Are these conditions also sufficient?

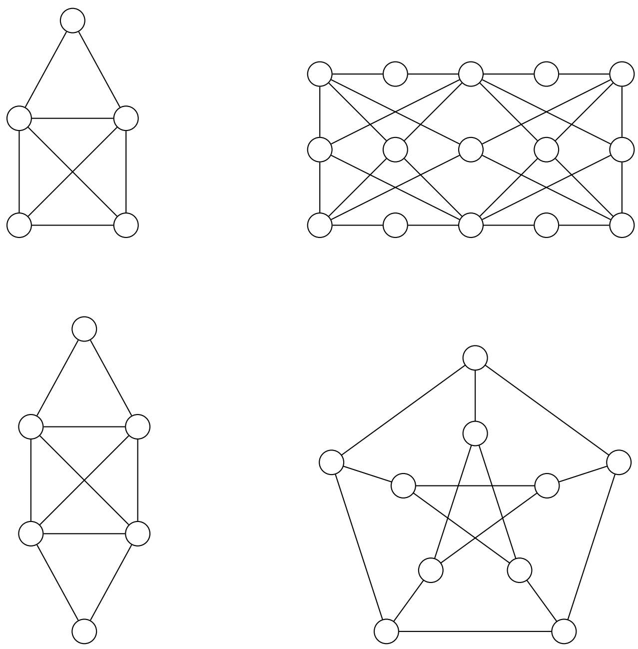

Figure 1: Eulerian or not?

Homework

Exercise H1 (5 points)
Show: In any graph with at least two vertices, there are at least two vertices with the same degree.

Exercise H2 (5 points)
For a graph $G = (V, E)$, the graph $\overline{G} = (V, \binom{V}{2} \setminus E)$ is called the *complementary graph* of G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G.

Show: One of G and $\overline{G}$ is connected.

Exercise H3 (10 points)
Show that for a graph $G = (V, E)$ with $n \geq 2$ vertices the following are equivalent:

(i) G is a tree.

(ii) G is connected and contains $n - 1$ edges.

(iii) G contains $n - 1$ edges, but no cycle.

(iv) G is *minimally connected*, *i.e.,* G is connected but $G - e$ is not connected for any $e \in E$.

(v) G is *maximally acyclic*, *i.e.,* G is acyclic but $G + e$ contains a cycle for any $e \in \binom{V}{2} \setminus E$.

(vi) For each pair $u, v \in V$ of vertices, there is a unique $[u, v]$-path in G.