Topological Groups
7. Exercise Sheet

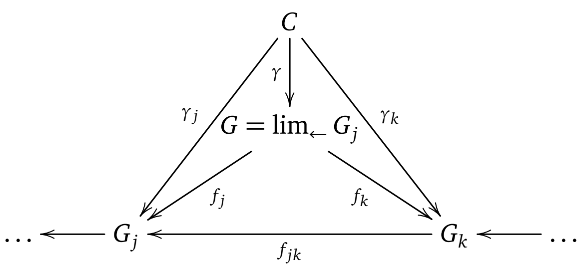
TECHNISCHE UNIVERSITÄT DARMSTADT

Department of Mathematics
Andreas Mars

Summer Term 2013
8.7.2013

Groupwork

Exercise G1 (Projective Limits)
Let $\mathscr{P} = \{f_{jk} \colon G_k \to G_j\}$ be a projective system of topological groups with limit maps $f_j \colon \lim_{\leftarrow} G_j = G \to G_j$. A **cone** over $\mathscr{P}$ is a topological group C together with morphisms $\gamma_j \colon C \to G_j$ of topological groups, such that for $j \leq k$ the identity $\gamma_j = f_{jk} \circ \gamma_k$ holds. In other words, the following diagram commutes:



(a) Show that (or rather: Convince yourself that ...): The group G with the limit maps $f_j \colon G \to G_j$ is a cone over $\mathscr{P}$.

(b) Show that with $C := \{1\}$ and the obvious maps γ_j we obtain a cone over $\mathscr{P}$.

(c) Prove the following universal property of the projective limit: If $\{\gamma_j \colon C \to G_j\}$ is a cone over $\mathscr{P}$, then there exists a unique morphism $\gamma \colon C \to G = \lim_{\leftarrow} G_j$ such that $\gamma_j = f_j \circ \gamma$.

Exercise G2 (Compact Lie Groups)
Show that:

(a) Every finite discrete group is a compact Lie group.

(b) A finite direct product of compact Lie groups is a compact Lie group. What about infinite direct products?

(c) A closed subgroup of a compact Lie group is a compact Lie group.

Exercise G3 (Divisibility)
Let A be an abelian group. Show that the following are equivalent:

(a) A is divisible.

(b) For every $a \in A$ there exists a homomorphism $f : \mathbb{Q} \to A$ such that $f(1) = a$.