Topological Groups
5. Exercise Sheet

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Groupwork

Exercise G1 (Warming Up)
Prove or disprove: The map $\phi \mapsto \int_0^1 \phi(x)dx$ is a Haar-integral on $\mathbb{R}$.

Exercise G2 (Unimodular Groups)
Let G be the subgroup of upper triangular matrices $\mathrm{SL}_2(\mathbb{R})$. Show that G is not unimodular.

Exercise G3 (Applications of the Haar-integral)
Let G be locally compact and let λ be a Haar-integral on G. Consider the space $C_c^{\mathbb{C}}(G) := \{f + ih \mid f, h \in C_c(G)\}$ and the natural extension λ to this space. Show that:

The map

$$\langle \cdot, \cdot \rangle : C_c^{\mathbb{C}}(G) \times C_c^{\mathbb{C}}(G) \to \mathbb{C}$$
$$(f, h) \mapsto \lambda(f\overline{h})$$

is a unitary scalar product on $C_c^{\mathbb{C}}(G)$ and the transformation $f \mapsto f_g$ is unitary for every $g \in G$.

Conclude that the completion of the space $(C_c^{\mathbb{C}}(G), \langle \cdot, \cdot \rangle)$ is a Hilbert space and G admits a unitary representation on this space.

Exercise G4 (Character Groups)
Compute the character group of $\mathbb{T}$.