Topological Groups
4. Exercise Sheet

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Groupwork

Exercise G1 (Lebesgue measure)
Show that the Lebesgue measure on $\mathbb{R}^n$ is a Haar measure.

Exercise G2 (Haar measures)
Let G be a finite group. Find all Haar measures on G.
What happens if we assume G to be discrete, but possibly infinite (for instance $G = \mathbb{Z}$)?

Exercise G3 (Some Group Theory)
Let G be a group, A an abelian group and let $f : G \to A$ be a group homomorphism. Show that: $[G, G] \le \ker(f)$.
Conclude: If G, A are topological groups, A is Hausdorff and f continuous, then it follows that $\overline{[G, G]} \le \ker(f)$.

Exercise G4 (Locally Compact Groups)
Prove or give a counterexample: A direct product of a family of locally compact groups is locally compact.

Exercise G5 (Existence of Functions)
Let G be locally compact and Hausdorff. Show that for $U \in \mathfrak{U}$ there exists a function $0 \neq f \in C_c^+(G)$ with $\operatorname{supp}(f) \subseteq U$.
Hint: Show that every locally compact group is $T_{3\frac{1}{2}}$, i.e. a point and a closed set may be separated by a continuous function (this requires some work). Use this fact with a suitable point and a suitable closed subset of G.