Zermelo-Fraenkel axioms of Set Theory

with foundation and axiom of choice

extensionality

(EXT) $\forall x \forall y \big(\forall z (z \in x \leftrightarrow z \in y) \to x = y\big)$

separation/comprehension (schema of axioms, one for each $\varphi(\mathbf{x}, z)$)

(SEP) $\forall \mathbf{x} \forall x \exists y \forall z \big(z \in y \leftrightarrow (z \in x \wedge \varphi(\mathbf{x}, z))\big)$

pair set

(PAIR) $\forall x \forall y \exists u \forall z \big(z \in u \leftrightarrow (z = x \vee z = y)\big)$

union

(UNION) $\forall x \exists y \forall z \big(z \in y \leftrightarrow \exists u (u \in x \wedge z \in u)\big)$

power set

(POWER) $\forall x \exists y \forall z \big(z \in y \leftrightarrow \forall u (u \in z \to u \in x)\big)$

infinity

(INF) $\exists x \big(\emptyset \in x \wedge \forall y (y \in x \to y \cup \{y\} \in x)\big)$

replacement (schema of axioms, one for each $\varphi(\mathbf{x}, u, v)$)

(REP) $$\forall \mathbf{x} \forall x \left[\begin{array}{l} \forall u \big(u \in x \to \exists^{=1} v \varphi(\mathbf{x}, u, v)\big) \\ \to \exists y \big(\forall v (v \in y \leftrightarrow \exists u (u \in x \wedge \varphi(\mathbf{x}, u, v)))\big) \end{array} \right]$$

foundation

(FOUND) $\forall x \big(\neg\, x = \emptyset \to \exists y \big(y \in x \wedge y \cap x = \emptyset\big)\big)$

axiom of choice

(AC) $$\forall x \left[\begin{array}{l} \big(\neg\, \emptyset \in x \wedge (\forall u \forall v (u \in x \wedge v \in x \wedge \neg u = v) \to u \cap v = \emptyset)\big) \\ \to \exists y \forall u \big(u \in x \to \exists z\; y \cap u = \{z\}\big) \end{array} \right]$$

In the FO-formulae of the axiom schemas we write $\mathbf{x}$ for a tuple of free variables; their components and all the other variable symbols are pairwise distinct; the formulae φ in the axiom schemas are arbitrary FO($\{\in\}$)-formulae (with free variables among the indicated);

$\exists^{=1} x \varphi$ is shorthand for $\exists x(\varphi \wedge \forall y (\varphi\frac{y}{x} \to x = y))$, which asserts the existence of a unique x satisfying φ;

"inofficial" terms like $\emptyset, u \cap v, u \cup v, \{z\}$ are to be treated as abbreviations for the use of agreed FO($\{\in\}$) definitions; similarly for "inofficial" relations like $x \subseteq y$. Some elimination examples:

$\emptyset \in x$	$\exists y \big(y \in x \wedge \forall z\, \neg\, z \in y\big)$
$x = \{z\}$	$\forall u (u \in x \leftrightarrow u = z)$
$x = u \cap v$	$\forall z \big(z \in x \leftrightarrow (z \in u \wedge z \in v)\big)$
$x \subseteq y$	$\forall z (z \in x \to z \in y)$