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Aus Vielfalt wird Einheit

Das Internet - ein Hort der Meinungsvielfalt? Theoretisch schon. Doch in der Realität herrscht Einförmigkeit der Ansichten. Das liegt zum einen an uns selbst, zum anderen an den Filtern von Amazon, Google, Facebook & Co. Von Carsten Görig

-------------- Carsten Görig -------

full text here:

http://www.stern.de/digital/online/meinungsbildung-im-internet-aus-vielfalt-wird-einheit-1664369.html

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Zitat, wörtlich:

Neue Techniken führen dazu, dass die Menschen zueinanderfinden. Zugang zu allen Informationen führt dazu, dass wir uns umfassend informieren, Verständnis für andere Meinungen entwickeln und unsere Standpunkte ausgewogener werden - das zumindest propagieren die Verfechter sozialer Netzwerke und Suchmaschinen wie Google. Das Netz ist der große Heilsbringer. Viele Studien zeigen allerdings: Das Gegenteil ist der Fall. Wir denken immer eindimensionaler. Das Netz gibt uns unzählige Möglichkeiten, die eigene Meinung zu stärken und zu verfestigen. Gleichzeitig gibt es uns sehr viele Möglichkeiten, anderen Meinungen auszuweichen. Das ist ein dem System inliegendes Problem: Wenn man es jemandem bequem machen möchte, sucht man ihm Sachen heraus, die er kennt, an die er gewöhnt ist. Und das Ziel der neuen Dienste ist es ja, es dem Menschen im Internet so bequem wie möglich zu machen.

1971 veröffentlicht der amerikanische Wissenschaftler Thomas Schelling eine Studie, die den Titel "Models of Segregation" trägt (Modelle der Trennung), wobei er sich mit dem Begriff der Trennung auf Rassentrennung in Städten bezieht. Auf einem Spielbrett legt er dar, wie selbst eine nur geringe Vorliebe, sich mit Menschen der eigenen Hautfarbe zu umgeben, zu einer vollständigen Trennung von Wohngebieten führen kann. Ein Prozess, der in Städten Jahre und Jahrzehnte dauern, woanders aber deutlich schneller verlaufen kann.

Wir suchen Zustimmung

Dieses Modell lässt sich auf andere Formen des menschlichen Zusammenlebens übertragen, auch auf das Internet, auf unsere Surf-Gewohnheiten. Wir neigen dazu, Dinge zu suchen, die uns näher sind, die uns in unserer Meinung bestärken. Und das tun wir auch im Netz. Mit jedem Klick, mit jeder Seite, die wir uns anschauen, erziehen wir die Suchmaschinen, uns mehr von dem zu zeigen, was wir mögen, weniger von dem, was wir nicht mögen. So kommen wir immer weniger in Kontakt mit Meinungen, die wir nicht so gerne sehen oder lesen. Es ist ein individueller Prozess, bei dem sich die Suchmaschine unseren Bedürfnissen anpasst, oder vielmehr dem, was sie als unsere Bedürfnisse errechnet. Mit dem Ergebnis, dass die Maschine uns irgendwann nur noch die Seiten oben anzeigt, die sie für uns als einzelne Person für wichtig hält.

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Doch es ist die technische Idee von der Funktion des Gehirns, die in Firmen wie Google oder Facebook vorherrscht. Und in der hat der Zufall keinen Platz. Schon bevor wir einen Begriff eingeben, möchte Google uns die Antwort auf das geben, was wir suchen. Das ist eines der fernen Ziele der Google-Gründer. Doch das funktioniert nur, wenn sie uns genau kennen, wenn sie unsere Gewohnheiten auswerten und wir innerhalb dieser handeln. Und genau deshalb steuern sie uns weiter in eine Ecke, aus

der wir nur schwer wieder herauskommen, und verfestigen und bilden damit unsere Meinung.

Learning and Teaching with Maple:

Steps:
- some necessary prior knowledge
- understanding the basic problem
- understanding the basic solution algorithm
- Problem 1: nodes without successors / webpages without links
- Problem 2: importance-sinks
- Problem 3: Runtime, Power-Method

Supposed prior Knowledge:

1.) Discrete Probabilities
2.) Matrix operations: sums, matrix-matrix multiplication, matrix-vector multiplication

The google problem, the basic problem:

given is

- a library with 25 billion documents
- no centralized organisation
- no librarians
- anyone can add documents

You are interested in information. You only know some keywords, and a further complication is:

Google claims **more then 25 billion indexed pages**. **95% of the text** in the Web is composed of **only some 1,000 words**. How can we distinguish the important pages from the unimportant ones?

Impossible?

```
> restart; with(LinearAlgebra);
```

[&x, *Add*, *Adjoint*, *BackwardSubstitute*, *BandMatrix*, *Basis*, *BezoutMatrix*, *BidiagonalForm*, *BilinearForm*, *CARE*, *CharacteristicMatrix*, *CharacteristicPolynomial*, *Column*, *ColumnDimension*, *ColumnOperation*, *ColumnSpace*, *CompanionMatrix*, *ConditionNumber*, *ConstantMatrix*, *ConstantVector*, *Copy*, *CreatePermutation*, *CrossProduct*, *DARE*, *DeleteColumn*, *DeleteRow*, *Determinant*, *Diagonal*, *DiagonalMatrix*, *Dimension*, *Dimensions*, *DotProduct*, *EigenConditionNumbers*, *Eigenvalues*, *Eigenvectors*, **(1)** *Equal*, *ForwardSubstitute*, *FrobeniusForm*, *GaussianElimination*, *GenerateEquations*, *GenerateMatrix*, *Generic*, *GetResultDataType*, *GetResultShape*, *GivensRotationMatrix*, *GramSchmidt*, *HankelMatrix*, *HermiteForm*, *HermitianTranspose*, *HessenbergForm*, *HilbertMatrix*, *HouseholderMatrix*, *IdentityMatrix*, *IntersectionBasis*, *IsDefinite*, *IsOrthogonal*, *IsSimilar*, *IsUnitary*, *JordanBlockMatrix*, *JordanForm*, *KroneckerProduct*, *LA_Main*, *LUDecomposition*, *LeastSquares*, *LinearSolve*, *LyapunovSolve*, *Map*, *Map2*, *MatrixAdd*, *MatrixExponential*, *MatrixFunction*, *MatrixInverse*, *MatrixMatrixMultiply*, *MatrixNorm*, *MatrixPower*, *MatrixScalarMultiply*, *MatrixVectorMultiply*, *MinimalPolynomial*, *Minor*, *Modular*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *PopovForm*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *StronglyConnectedBlocks*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

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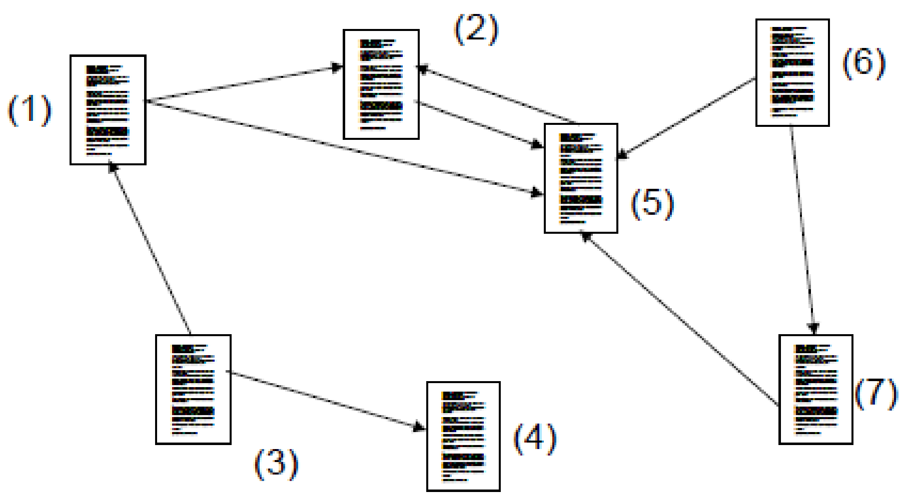
The basic algorithm: The heart of the google software is the PageRank algorithm.



Let P be a web page.

We call Imp(P) the importance of P.

Let P_j have L_j many outgoing links to other pages.
If P_i is such a target-page, P_j will pass $1/L_j$ „importance" to P_i.

Let B_i be the set of pages linking to P_i. Then the importance relation between a page and its neighbours is as follows:

$$\text{Imp}(P_i) = \sum_{P_j \in B_i} \frac{Imp(P_j)}{L_j}; \# \; ?? \;\; \textit{chicken vs. egg problem}$$

Next step: define a matrix H = (h_{ij}) with

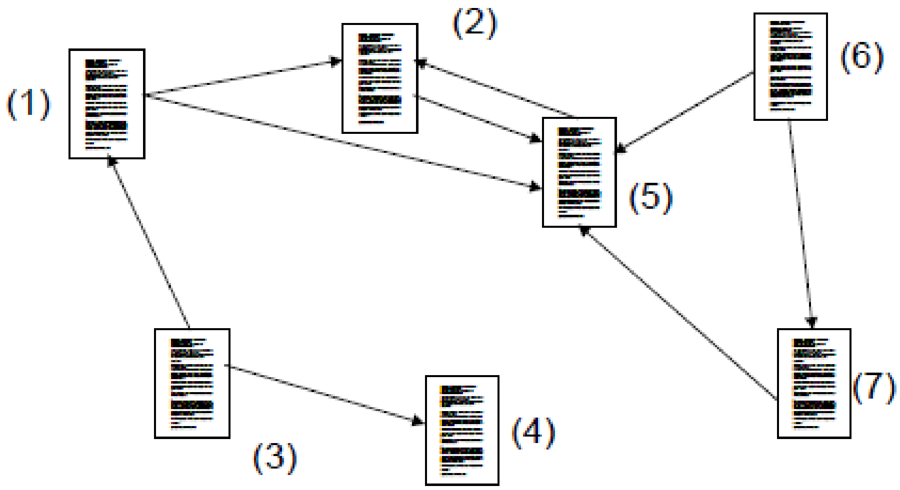
$$h_{ij} := \begin{cases} \frac{1}{L_j}, \; \textbf{if} \; P_j \in B_i \\ 0, \; otherwise \end{cases} \quad \text{(what does i get from j?)}$$

Example: $B_7 = \{6\}$ -> $L_6 = 2$ --> line 7: (0,0,0,0,0,1/2,0)
$B_5 = \{1,2,6,7\}$ -> $L_1 = 2, L_2 = 1, \; L_6 = 2, \; L_7 = 1$ --> line 5: (1/2,1,0,0,0,0,1/2,1)

Then H =

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix} \; \textbf{and} \; \textit{with a vector Imp of pageranks, e.g.} \; Imp = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \; \textit{it is valid} : Imp = H \cdot Imp$$

> $H := \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix} : Imp := \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} : \#$



> *Imp*, *H.Imp*; # *check that Imp = H·Imp*

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(2)}$$

Let **A** be a quadratic matrix with m rows and m columns.
The task to find a number λ and a vector x (≠ 0), such that

$$Ax = \lambda x,$$

is called Eigenvalue problem.

We saw:

a correct PageRank assignment can be interpreted as the eigenvector Imp of a matrix H with eigenvalue 1, such that Imp = 1*H*Imp:

Problem 1: Unfortunately, H contains so called dangling nodes, i.e. nodes without sucessors.

Consequence: zero-columns => H not stochastic => possibly no stationary solution

```
> (eigenvalues, eigenvectors) := LinearAlgebra[Eigenvectors](H);
```

$$eigenvalues, eigenvectors := \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

v is a vector of eigenvalues, e the matrix of all eigenvectors. The i-th eigenvalue corresponds to the i-th eigenvector.
-> good luck. Matrix H has a solution.

Control:

```
> Imp := Column(eigenvectors, [2]); #remember: we are looking for an Imp with Imp = H*Imp
```

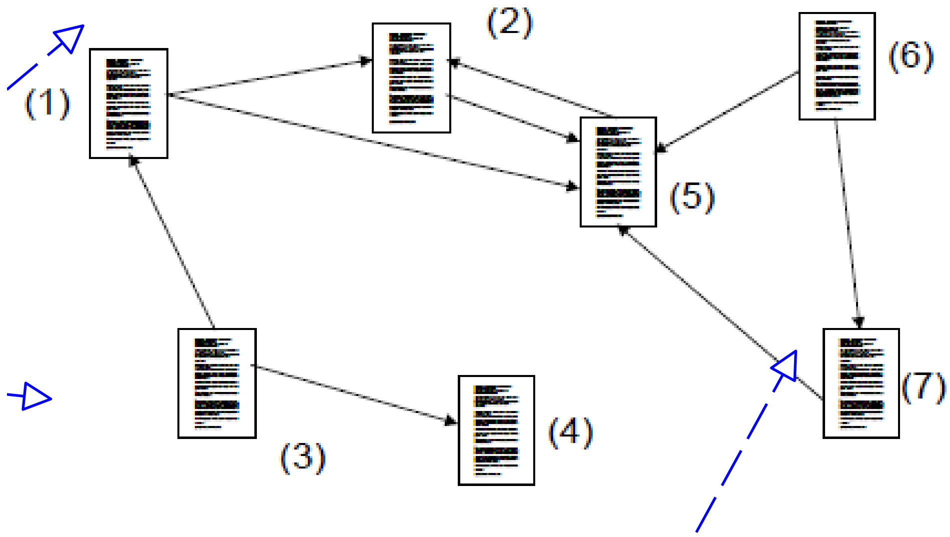
$$Imp := \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (4)$$

```
> H.Imp, Imp
```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

Now, let A be the matrix whose entries are all zero except for the columns of the dangling nodes, in which each entry is 1/n,
n being the number of nodes. Let S := H + A.

$$> A := \begin{bmatrix} 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \end{bmatrix} : S := H + A : \quad \#``$$



> $H, A, S;$

$$\left[\begin{array}{ccccccc} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{array}\right], \left[\begin{array}{ccccccc} 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \end{array}\right], \left[\begin{array}{ccccccc} 0 & 0 & \frac{1}{2} & \frac{1}{7} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{7} & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{7} & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & \frac{1}{7} & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & \frac{1}{2} & 0 \end{array}\right] \qquad (6)$$

Now, for each column of S is valid that the entries of each column sum up to one. This guarantees the existence of a stationary vector. (*No proof here, but there exists a Theorem.*) S is called a "stochastic matrix".

--> New interpretation: there is a random surfer on the web. Which portion of time will he spend
in which node, if she decides her next jump concerning transition-probabilities as they are described in the matrix S?

Let us take a look at the solution with the help of matrix S:

> $(eigenvalues, eigenvectors) := LinearAlgebra[Eigenvectors](S);$

$$eigenvalues, eigenvectors := \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ \frac{1}{14}+\frac{1}{14}\sqrt{15} \\ \frac{1}{14}-\frac{1}{14}\sqrt{15} \end{array}\right], \left[\left[\left[0, -2, 0, 0, 0, \right.\right.\right. \tag{7}$$

$$-\frac{1}{2025}\,\frac{1}{\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14}+\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^3\right.\right.$$

$$\left.\left.-166\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}-\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right),$$

$$\frac{1}{2025}\,\frac{1}{\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3\right.\right.$$
$$\left.\left.\left.-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\right],$$

$$\left[-1, 1, 0, 1, -1, -\frac{1}{2025}\left(\left(-4166\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^3-246\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^2\right.\right.\right.$$

$$\left.\left.+\frac{21}{2}-\frac{5}{2}\sqrt{15}-8519\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^4+32214\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^5\right)\sqrt{15}\right)$$

$$\Big/\left(\left(\frac{8}{7}+\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}+\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^2\right), \frac{1}{2025}\left(\left(\right.\right.$$

$$-4166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3-246\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2+\frac{21}{2}+\frac{5}{2}\sqrt{15}$$

$$-8519\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^4+32214\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^5\right)\sqrt{15}\Big)\Big/\Big(\Big(\frac{8}{7}$$

$$-\frac{1}{7}\sqrt{15}\Big)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\Big)\Big],$$

$$\Big[0, 0, 0, 0, 0,$$

$$-\frac{2}{2025}\Big(\Big(2301\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^3-166\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}$$

$$-\frac{65}{14}\sqrt{15}\Big)\sqrt{15}\Big)\Big/\Big(\left(\frac{8}{7}+\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}+\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)\Big),$$

$$\frac{2}{2025}\Big(\Big(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}$$

$$+\frac{65}{14}\sqrt{15}\Big)\sqrt{15}\Big)\Big/\Big(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)\Big)$$

$$\Big],$$

$$\Big[0, 0, 0, 0, 0,$$

$$-\frac{14}{2025}\frac{1}{\left(\frac{8}{7}+\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}+\frac{1}{14}\sqrt{15}\right)}\Big(\Big(2301\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^3$$

$$-166\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}-\frac{65}{14}\sqrt{15}\Big)\sqrt{15}\Big),$$

$$\frac{14}{2025}\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Big(\Big(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\Big)\sqrt{15}\Big)\Big],$$

$$\left[0, 1, 0, 1, 1, \frac{1}{15} \frac{177 \left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)^2 - \frac{3}{14} + \frac{11}{14}\sqrt{15}}{\left(\frac{8}{7} + \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} + \frac{1}{14}\sqrt{15}\right)},\right.$$

$$\left.\frac{1}{15} \frac{177 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{3}{14} - \frac{11}{14}\sqrt{15}}{\left(\frac{8}{7} - \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)}\right],$$

$$\left[0, 0, 0, 0, 0, \frac{2\left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)}{\frac{8}{7} + \frac{1}{7}\sqrt{15}}, \frac{2\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)}{\frac{8}{7} - \frac{1}{7}\sqrt{15}}\right],$$

$$\left[1, 0, 0, 0, 0, 1, 1\right]\Big]$$

> *Imp* := *Column*(*eigenvectors*, [7]), *S.Imp*;
    # eigenvectors build a matrix. We need the last column!

$$Imp := \qquad \textbf{(8)}$$

$$\left[\frac{1}{2025} \frac{1}{\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 \left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)} \left(\left(2301 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^3 - 166 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{247}{14} + \frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\right],$$

$$\left[\frac{1}{2025} \left(\left(-4166 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^3 - 246 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 + \frac{21}{2} + \frac{5}{2}\sqrt{15} - 8519 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^4 + 32214 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^5\right)\sqrt{15}\right) \Big/ \left(\left(\frac{8}{7} - \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2\right)\right],$$

$$\left[\frac{2}{2025} \left(\left(2301 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^3 - 166 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{247}{14} + \frac{65}{14}\sqrt{15}\right)\sqrt{15}\right) \Big/ \left(\left(\frac{8}{7} - \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)\right)\right],$$

$$\left[\frac{14}{2025}\;\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3 - 166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\right],$$

$$\left[\frac{1}{15}\;\frac{177\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{3}{14}-\frac{11}{14}\sqrt{15}}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\right],$$

$$\left[\frac{2\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)}{\frac{8}{7}-\frac{1}{7}\sqrt{15}}\right],$$

$$\left[1\right]\Bigg], \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\end{bmatrix}$$

If Imp is a solution to our problem, then also 1/2*Imp is a solution: H * (1/2 * Imp) = 1/2 * Imp

> $\frac{1}{2}.Imp;$

$$\Bigg[\left[\frac{1}{4050}\;\frac{1}{\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3 - 166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\right], \tag{9}$$

$$\left[\frac{1}{4050}\left(\left(-4166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3-246\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2+\frac{21}{2}+\frac{5}{2}\sqrt{15} - 8519\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^4+32214\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^5\right)\sqrt{15}\right)\Big/\left(\left(\frac{8}{7} - \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\right)\right],$$

$$\left[\frac{1}{2025}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}\right.\right.\right.$$

$$+ \frac{65}{14}\sqrt{15}\Big)\sqrt{15}\Big) \Big/ \left(\left(\frac{8}{7} - \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)\right)$$

$$\Big],$$

$$\left[\frac{7}{2025}\,\frac{1}{\left(\frac{8}{7} - \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^3 - 166\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{247}{14} + \frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\right],$$

$$\left[\frac{1}{30}\,\frac{177\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{3}{14} - \frac{11}{14}\sqrt{15}}{\left(\frac{8}{7} - \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)}\right],$$

$$\left[\frac{\frac{1}{14} - \frac{1}{14}\sqrt{15}}{\frac{8}{7} - \frac{1}{7}\sqrt{15}}\right],$$

$$\left[\frac{1}{2}\right]\Big]\Big]$$

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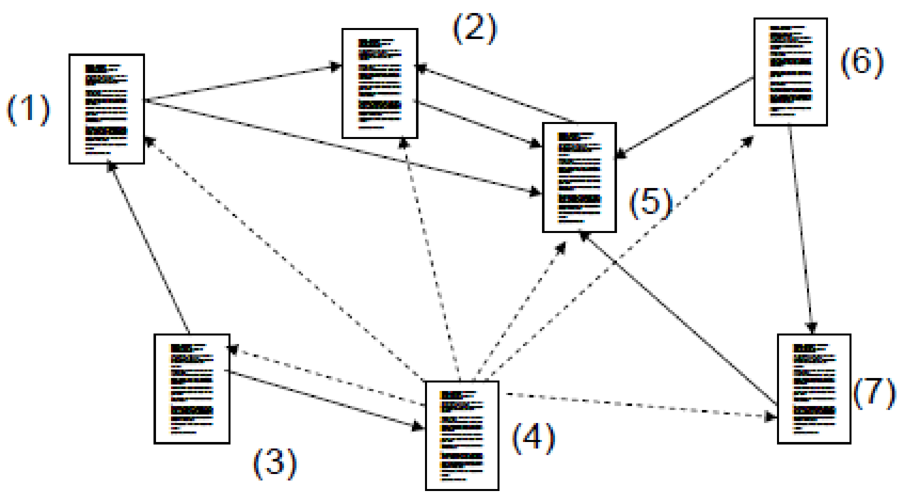

Unfortunately, there is **Problem 2:**

The nodes (2) and (5) are "importance sinks".
-> In the graph, you see that the random walker is trapped
-> The graph is sayed to be "not strongly connected". It does not exists a path from any node to any other node.

-> The matrix is not "irreducible", i.e. S can be written in block form: $S = \begin{bmatrix} * & 0 \\ * & * \end{bmatrix}$.

Strongly connected graphs produce irreducible matrices.
(*No proof here, but there exists a Theorem.*)

> $E := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix};$

$$E := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{10}$$

> $G := \frac{85}{100} \cdot S + \left(1 - \frac{85}{100}\right) \cdot \frac{1}{7} \cdot E;$ # $\frac{1}{7}$ *because we have* 7 *nodes* - - $\rightarrow$ *there is a bit connection between all pairs of pages*

(11)

$$G := \begin{bmatrix} \frac{3}{140} & \frac{3}{140} & \frac{25}{56} & \frac{1}{7} & \frac{3}{140} & \frac{3}{140} & \frac{3}{140} \\ \frac{25}{56} & \frac{3}{140} & \frac{3}{140} & \frac{1}{7} & \frac{61}{70} & \frac{3}{140} & \frac{3}{140} \\ \frac{3}{140} & \frac{3}{140} & \frac{3}{140} & \frac{1}{7} & \frac{3}{140} & \frac{3}{140} & \frac{3}{140} \\ \frac{3}{140} & \frac{3}{140} & \frac{25}{56} & \frac{1}{7} & \frac{3}{140} & \frac{3}{140} & \frac{3}{140} \\ \frac{25}{56} & \frac{61}{70} & \frac{3}{140} & \frac{1}{7} & \frac{3}{140} & \frac{25}{56} & \frac{61}{70} \\ \frac{3}{140} & \frac{3}{140} & \frac{3}{140} & \frac{1}{7} & \frac{3}{140} & \frac{3}{140} & \frac{3}{140} \\ \frac{3}{140} & \frac{3}{140} & \frac{3}{140} & \frac{1}{7} & \frac{3}{140} & \frac{25}{56} & \frac{3}{140} \end{bmatrix} \quad \textbf{(11)}$$

> $(eigenvalues, eigenvectors) := LinearAlgebra[Eigenvectors](G)$:

> $eigenvalues$;

$$\begin{bmatrix} \frac{17}{280} + \frac{17}{280}\sqrt{15} \\ \frac{17}{280} - \frac{17}{280}\sqrt{15} \\ 1 \\ -\frac{17}{20} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(12)}$$

> $Imp := Column(eigenvectors, [1]), G.Imp$;

$$Imp := \left[\left[\left(2107\left(\frac{489874635527}{280} + 14603738560000\left(\frac{17}{280} + \frac{17}{280}\sqrt{15}\right)^5 - 8632290344000\left(\frac{17}{280} + \frac{17}{280}\sqrt{15}\right)^4 - 2632160925200\left(\frac{17}{280} + \frac{17}{280}\sqrt{15}\right)^3 + 136619160740\left(\frac{17}{280} + \frac{17}{280}\sqrt{15}\right)^2 + \frac{441606179207}{280}\sqrt{15}\right)\right) \Big/ \left(\left(\frac{136}{7} + \frac{17}{7}\sqrt{15}\right)\left(-\frac{165}{14} + \frac{17}{14}\sqrt{15}\right)\left(\frac{67439}{280} + \frac{15079}{280}\sqrt{15}\right)\left(-\frac{21913}{14} + \frac{23069}{14}\sqrt{15}\right)\left(\frac{13651}{7} + \frac{14722}{7}\sqrt{15}\right)\left(\frac{11}{2} + \frac{17}{2}\sqrt{15}\right)\right)\right], \quad \textbf{(13)}$$

$$\left[\left(\frac{30740848087863}{280} + 1825467320000000\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^5 + 1039538356808000\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^4 + 27951509058400\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^3 - 23101991103720\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^2 - \frac{28392734851897}{280}\sqrt{15}\right) \Big/ \left(\left(\frac{136}{7}+\frac{17}{7}\sqrt{15}\right)\left(-\frac{165}{14}+\frac{17}{14}\sqrt{15}\right)\left(\frac{67439}{280}+\frac{15079}{280}\sqrt{15}\right)\left(-\frac{21913}{14}+\frac{23069}{14}\sqrt{15}\right)\left(\frac{13651}{7}+\frac{14722}{7}\sqrt{15}\right)\right)\right],$$

$$\left[\left(2\left(\frac{5995613941}{280}+\frac{1222143141}{280}\sqrt{15} - 1769549720\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^2 - 3728279200\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^3 + 15975528000\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^4\right)\right) \Big/ \left(\left(\frac{136}{7}+\frac{17}{7}\sqrt{15}\right)\left(-\frac{165}{14}+\frac{17}{14}\sqrt{15}\right)\left(\frac{67439}{280}+\frac{15079}{280}\sqrt{15}\right)\left(-\frac{21913}{14}+\frac{23069}{14}\sqrt{15}\right)\right)\right],$$

$$\left[-\frac{1}{20}\left(-\frac{2563430}{7}-\frac{769029}{7}\sqrt{15}+12113200\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^2 + 60512000\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^3\right) \Big/ \left(\left(\frac{136}{7}+\frac{17}{7}\sqrt{15}\right)\left(-\frac{165}{14}+\frac{17}{14}\sqrt{15}\right)\left(\frac{67439}{280}+\frac{15079}{280}\sqrt{15}\right)\right)\right],$$

$$\left[\frac{2\left(-\frac{425}{28}+\frac{51}{28}\sqrt{15}+1800\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)^2\right)}{\left(\frac{136}{7}+\frac{17}{7}\sqrt{15}\right)\left(-\frac{165}{14}+\frac{17}{14}\sqrt{15}\right)}\right],$$

$$\left[\frac{40\left(\frac{17}{280}+\frac{17}{280}\sqrt{15}\right)}{\frac{136}{7}+\frac{17}{7}\sqrt{15}}\right],$$

$$\left[1\right]\Bigg]\Bigg], \Bigg[$$

$$\left[\frac{1}{94500}\,\frac{1}{\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3\right.\right.\right.$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{2}-\frac{247}{14}+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)+\frac{1}{94500}\left(\left(-4166\left(\frac{1}{14}\right.\right.\right.$$

$$-\frac{1}{14}\sqrt{15}\right)^{3}-246\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{2}+\frac{21}{2}+\frac{5}{2}\sqrt{15}-8519\left(\frac{1}{14}\right.$$

$$-\frac{1}{14}\sqrt{15}\right)^{4}+32214\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{5}\right)\sqrt{15}\right)\Big/\left(\left(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}\right.\right.\right.$$

$$-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{2}\right)$$

$$+\frac{1}{2268}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{3}-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{2}-\frac{247}{14}\right.\right.$$

$$+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\Big/\left(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)\right)$$

$$+\frac{2}{2025}\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{3}\right.\right.$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{2}-\frac{247}{14}+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)$$

$$+\frac{1}{700}\frac{177\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{2}-\frac{3}{14}-\frac{11}{14}\sqrt{15}}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}+\frac{3}{70}\frac{\frac{1}{14}-\frac{1}{14}\sqrt{15}}{\frac{8}{7}-\frac{1}{7}\sqrt{15}}+\frac{3}{140}$$

],

$$\left[\frac{1}{4536}\frac{1}{\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Bigg(\Bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3\right.$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\Bigg)\sqrt{15}\Bigg)+\frac{1}{94500}\Bigg(\Bigg(-4166\Bigg(\frac{1}{14}$$

$$-\frac{1}{14}\sqrt{15}\Bigg)^3-246\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2+\frac{21}{2}+\frac{5}{2}\sqrt{15}-8519\Bigg(\frac{1}{14}$$

$$-\frac{1}{14}\sqrt{15}\Bigg)^4+32214\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^5\Bigg)\sqrt{15}\Bigg)\Bigg/\Bigg(\Bigg(\frac{8}{7}-\frac{1}{7}\sqrt{15}\Bigg)\Bigg(-\frac{13}{14}$$

$$-\frac{1}{14}\sqrt{15}\Bigg)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\Bigg)$$

$$+\frac{1}{47250}\Bigg(\Bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}$$

$$+\frac{65}{14}\sqrt{15}\Bigg)\sqrt{15}\Bigg)\Bigg/\Bigg(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)\Bigg)$$

$$+\frac{2}{2025}\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Bigg(\Bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\Bigg)\sqrt{15}\Bigg)$$

$$+\frac{61}{1050}\frac{177\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{3}{14}-\frac{11}{14}\sqrt{15}}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}+\frac{3}{70}\frac{\frac{1}{14}-\frac{1}{14}\sqrt{15}}{\frac{8}{7}-\frac{1}{7}\sqrt{15}}$$

$$+\frac{3}{140}\Bigg],$$

$$\Bigg[\frac{1}{94500}\,\frac{1}{\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Bigg(\Bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\Bigg)\sqrt{15}\Bigg)+\frac{1}{94500}\Bigg(\Bigg(-4166\Bigg(\frac{1}{14}$$

$$-\frac{1}{14}\sqrt{15}\Bigg)^3-246\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2+\frac{21}{2}+\frac{5}{2}\sqrt{15}-8519\Bigg(\frac{1}{14}$$

$$-\frac{1}{14}\sqrt{15}\Bigg)^4+32214\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^5\Bigg)\sqrt{15}\Bigg)\Bigg/\Bigg(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\Bigg(-\frac{13}{14}$$

$$-\frac{1}{14}\sqrt{15}\Bigg)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\Bigg)$$

$$+\frac{1}{47250}\Bigg(\Bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}$$

$$+\frac{65}{14}\sqrt{15}\Bigg)\sqrt{15}\Bigg)\Bigg/\Bigg(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)\Bigg)$$

$$+\frac{2}{2025}\,\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Bigg(\Bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\Bigg)\sqrt{15}\Bigg)$$

$$+\frac{1}{700}\,\frac{177\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{3}{14}-\frac{11}{14}\sqrt{15}}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}+\frac{3}{70}\,\frac{\frac{1}{14}-\frac{1}{14}\sqrt{15}}{\frac{8}{7}-\frac{1}{7}\sqrt{15}}+\frac{3}{140}$$

$$\Bigg],$$

$$\Bigg[\frac{1}{94500}\,\frac{1}{\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Bigg(\Bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\Bigg)\sqrt{15}\Bigg)+\frac{1}{94500}\Bigg(\Bigg(-4166\Bigg(\frac{1}{14}$$

$$-\frac{1}{14}\sqrt{15}\Bigg)^3-246\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2+\frac{21}{2}+\frac{5}{2}\sqrt{15}-8519\Bigg(\frac{1}{14}$$

$$-\frac{1}{14}\sqrt{15}\Bigg)^4+32214\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^5\Bigg)\sqrt{15}\Bigg)\Bigg/\Bigg(\Bigg(\frac{8}{7}-\frac{1}{7}\sqrt{15}\Bigg)\Bigg(-\frac{13}{14}$$

$$-\frac{1}{14}\sqrt{15}\Bigg)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\Bigg)$$

$$+\frac{1}{2268}\Bigg(\Bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}$$

$$+\frac{65}{14}\sqrt{15}\Bigg)\sqrt{15}\Bigg)\Bigg/\Bigg(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)\Bigg)$$

$$+\frac{2}{2025}\,\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Bigg(\Bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3$$

$$- 166 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{247}{14} + \frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)$$

$$+ \frac{1}{700}\frac{177\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{3}{14} - \frac{11}{14}\sqrt{15}}{\left(\frac{8}{7} - \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)} + \frac{3}{70}\frac{\frac{1}{14} - \frac{1}{14}\sqrt{15}}{\frac{8}{7} - \frac{1}{7}\sqrt{15}} + \frac{3}{140}$$

],

$$\Bigg[\frac{1}{4536}\frac{1}{\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^3\right.\right.$$

$$- 166\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{247}{14} + \frac{65}{14}\sqrt{15}\right)\sqrt{15}\right) + \frac{61}{141750}\left(\left(\right.\right.$$

$$-4166\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^3 - 246\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 + \frac{21}{2} + \frac{5}{2}\sqrt{15}$$

$$- 8519\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^4 + 32214\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^5\right)\sqrt{15}\right) \Big/ \left(\left(\frac{8}{7}\right.\right.$$

$$- \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2\right)$$

$$+ \frac{1}{47250}\left(\left(2301\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^3 - 166\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{247}{14}\right.\right.$$

$$+ \frac{65}{14}\sqrt{15}\right)\sqrt{15}\right) \Big/ \left(\left(\frac{8}{7} - \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)\right)$$

$$+\frac{2}{2025}\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3\right.\right.$$

$$\left.\left.-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)$$

$$+\frac{1}{700}\frac{177\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{3}{14}-\frac{11}{14}\sqrt{15}}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}+\frac{25}{28}\frac{\frac{1}{14}-\frac{1}{14}\sqrt{15}}{\frac{8}{7}-\frac{1}{7}\sqrt{15}}+\frac{61}{70}$$

],

$$\left[\frac{1}{94500}\frac{1}{\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3\right.\right.\right.$$

$$\left.\left.-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)+\frac{1}{94500}\left(\left(-4166\left(\frac{1}{14}\right.\right.\right.$$

$$\left.-\frac{1}{14}\sqrt{15}\right)^3-246\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2+\frac{21}{2}+\frac{5}{2}\sqrt{15}-8519\left(\frac{1}{14}\right.$$

$$\left.\left.\left.-\frac{1}{14}\sqrt{15}\right)^4+32214\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^5\right)\sqrt{15}\right)\Big/\left(\left(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}\right.\right.\right.$$

$$\left.\left.-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\right)$$

$$+\frac{1}{47250}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}\right.\right.$$

$$+\frac{65}{14}\sqrt{15}\Big)\sqrt{15}\Big)\Big/\left(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)\right)$$

$$+\frac{2}{2025}\,\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Bigg(\bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\bigg)\sqrt{15}\Bigg)$$

$$+\frac{1}{700}\,\frac{177\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{3}{14}-\frac{11}{14}\sqrt{15}}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}+\frac{3}{70}\,\frac{\frac{1}{14}-\frac{1}{14}\sqrt{15}}{\frac{8}{7}-\frac{1}{7}\sqrt{15}}+\frac{3}{140}$$

],

$$\Bigg[\frac{1}{94500}\,\frac{1}{\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Bigg(\bigg(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3$$

$$-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\bigg)\sqrt{15}\Bigg)+\frac{1}{94500}\Bigg(\bigg(-4166\Big(\frac{1}{14}$$

$$-\frac{1}{14}\sqrt{15}\Big)^3-246\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2+\frac{21}{2}+\frac{5}{2}\sqrt{15}-8519\Big(\frac{1}{14}$$

$$-\frac{1}{14}\sqrt{15}\Big)^4+32214\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^5\bigg)\sqrt{15}\Bigg)\Big/\bigg(\Big(\frac{8}{7}-\frac{1}{7}\sqrt{15}\Big)\Big(-\frac{13}{14}$$

$$-\frac{1}{14}\sqrt{15}\Big)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2\bigg)$$

$$+\frac{1}{47250}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}\right.\right.$$

$$\left.\left.+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\Big/\left(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)\right)$$

$$+\frac{2}{2025}\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^3\right.\right.$$

$$\left.\left.-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{247}{14}+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)$$

$$+\frac{1}{700}\frac{177\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{3}{14}-\frac{11}{14}\sqrt{15}}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}+\frac{25}{28}\frac{\frac{1}{14}-\frac{1}{14}\sqrt{15}}{\frac{8}{7}-\frac{1}{7}\sqrt{15}}+\frac{3}{140}$$

$$\Big]\Big]$$

>

Problem 3: Runtime, Power method

The next question is, how we can compute the solution as fast as possible. The idea of the so called power method is to use the fact that under certain circumstances the sequence Imp^0 = a and Imp^(k+1) = G*Imp^k
converges to the correct solution.
It will do so, if the matrix G is irreducible and stochastic.(*There is a Theorem, no proof here*)

> $Start := \left\langle \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7} \right\rangle;$

(14)

$$Start := \begin{bmatrix} \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \end{bmatrix} \tag{14}$$

> $G.Start;$

$$\begin{bmatrix} \frac{39}{392} \\ \frac{433}{1960} \\ \frac{19}{490} \\ \frac{39}{392} \\ \frac{79}{196} \\ \frac{19}{490} \\ \frac{39}{392} \end{bmatrix} \tag{15}$$

> $G^2.Start;$ # = (G.G).Start = G.(G.Start)

(16)

$$\begin{bmatrix} \frac{13717}{274400} \\ \frac{45923}{109760} \\ \frac{1839}{54880} \\ \frac{13717}{274400} \\ \frac{200103}{548800} \\ \frac{1839}{54880} \\ \frac{13717}{274400} \end{bmatrix} \quad \textbf{(16)}$$

> $seq(evalf(G^k.Start), k = 8..11);$

$$\begin{bmatrix} 0.03693512985 \\ 0.4239101952 \\ 0.02591652167 \\ 0.03693512985 \\ 0.4134513719 \\ 0.02591652167 \\ 0.03693512985 \end{bmatrix}, \begin{bmatrix} 0.03692807319 \\ 0.3930446478 \\ 0.02591355148 \\ 0.03692807319 \\ 0.4443440297 \\ 0.02591355148 \\ 0.03692807319 \end{bmatrix}, \begin{bmatrix} 0.03692595398 \\ 0.4192995509 \\ 0.02591269460 \\ 0.03692595398 \\ 0.4180971979 \\ 0.02591269460 \\ 0.03692595398 \end{bmatrix}, \begin{bmatrix} 0.03692533247 \\ 0.3969885859 \\ 0.02591243727 \\ 0.03692533247 \\ 0.4404105421 \\ 0.02591243727 \\ 0.03692533247 \end{bmatrix} \quad \textbf{(17)}$$

> $seq(evalf(G^k.Start), k = 100..103);$

$$\begin{bmatrix} 0.03692507018 \\ 0.4072408675 \\ 0.02591232995 \\ 0.03692507018 \\ 0.4301592621 \\ 0.02591232995 \\ 0.03692507018 \end{bmatrix}, \begin{bmatrix} 0.03692507018 \\ 0.4072408576 \\ 0.02591232995 \\ 0.03692507018 \\ 0.4301592720 \\ 0.02591232995 \\ 0.03692507018 \end{bmatrix}, \begin{bmatrix} 0.03692507018 \\ 0.4072408660 \\ 0.02591232995 \\ 0.03692507018 \\ 0.4301592636 \\ 0.02591232995 \\ 0.03692507018 \end{bmatrix}, \begin{bmatrix} 0.03692507018 \\ 0.4072408588 \\ 0.02591232995 \\ 0.03692507018 \\ 0.4301592707 \\ 0.02591232995 \\ 0.03692507018 \end{bmatrix} \quad \textbf{(18)}$$

>

Last but not least: what about matrices with 25 billion rows and columns?

→ Remember : S = H + A

$$therefore: G = 0.85 \cdot H + 0.85 \cdot A + \frac{(1 - 0.85)}{n} \cdot E$$

therefore : $Imp^{k+1} = G \cdot Imp^{k} = 0.85 \cdot H \cdot Imp^{k} + 0.85 \cdot A \cdot Imp^{k} + \frac{(1 - 0.85)}{n} \cdot E \cdot Imp^{k}$

now : *most entries of H are zero. The rows of A are all the same*, **and** *the rows of E are all the same.*

therefore : *In practice only about* 300 *billion operations.*

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