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Algebraic, Topological, and Physical Aspects of Computing

SS 2012, Exercise Sheet #10

EXERCISE 10:

a) Prove that the real Halting problem $\mathbb{H}$ is not BSS–decidable, where

$$\mathbb{H} \;:=\; \big\{\langle \mathbb{M}, \vec{x}\rangle : \text{BSS machine } \mathbb{M} \text{ terminates on input } \vec{x} \in \mathbb{R}^*\big\}$$

b) Prove that a non-empty real language $\mathbb{L} \subseteq \mathbb{R}^*$ is semi-decidable iff
it coincides with the range of a computable total real function $f : \mathbb{R}^* \to \mathbb{R}^*$.
Hint: Tarski's quantifier elimination.

c) Formalize the problem of finding eigenvalues of real symmetric matrices
and prove it BSS–incomputable.

d) A BSS–machine over $\mathbb{C}$ (rather than $\mathbb{R}$) can in each step read, store, print, add, subtract, multiply, and divide two complex numbers; it may branch based on the result of comparing two complex numbers for *equality* only.
Is the following function computable by such a machine? Prove!

$$\mathbb{C}^{n\times n} \;\ni\; A \;\Mapsto\; \{B \in \mathbb{C}^{n\times n} : \text{range}(B) = \text{range}(A)^{\perp}\}$$

e) Prove that $\mathbb{N}$ is BSS–decidable over $\mathbb{R}$. In what running time? Is it decidable over $\mathbb{C}$? Prove!

f) Prove the algebraic degree function $\deg : \mathbb{A} \to \mathbb{N}$ BSS–computable without constants.