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Algebraic, Topological, and Physical Aspects of Computing

SS 2012, Exercise Sheet #9

EXERCISE 9:
Call a total multifunction $f : X \rightrightarrows Y$ doubly Henkin-continuous iff the following holds:

$$\begin{pmatrix} \forall \varepsilon > 0 & \exists \delta > 0 \\ \forall x \in X & \exists y \in f(x) \end{pmatrix} \begin{pmatrix} \forall \varepsilon' > 0 & \exists \delta' > 0 \\ \forall x' \in B(x, \delta) & \exists y' \in f(x') \cap B(y, \varepsilon) \end{pmatrix}$$
$$\forall x'' \in B(x', \delta')\ \exists y'' \in f(x'') \cap B(y', \varepsilon')$$

a) Recall Exercise 7e) and prove that the inverse $\rho_{\mathbb{D}}^{-1}$ of the dyadic representation is doubly Henkin-continuous.

b) Prove that every computable multivalued total $f : [0,1] \rightrightarrows \mathbb{R}$ is doubly Henkin-continuous.

c) Is Example 3.5 in the script double Henkin-continuous?

d) Construct an example of a compact, total, doubly Henkin–continuous relation $\subseteq [0,1] \times [0,1]$ which is not (even relatively) computable.

e) Prove that the following function is computable in exponential time, but not in polynomial time — and oracles do not help:

$$f : (0,1] \ni x \mapsto 1/\ln(e/x) \in (0,1], \qquad f(0) = 0 \ .$$

f) Construct a function $f : [0,1] \to \mathbb{R}$ which is computable in doubly exponential time $2^{2^{\mathcal{O}(n)}}$ but not in single exponential time (even with the help of oracles).