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Algebraic, Topological, and Physical Aspects of Computing

SS 2012, Exercise Sheet #8

EXERCISE 8:
For a multivalued function $f :\subseteq X \rightrightarrows Y$, recall the notions of weak continuity, nonuniform weak continuity, and uniform weak continuity.

a) Let f be uniformly weakly continuous and suppose that f is pointwise compact in the sense that $f(x) \subseteq Y$ is compact for every $x \in X$. Then f is weakly continuous.

b) Let f be nonuniformly weakly continuous and $\mathrm{dom}(f)$ compact.
Then f is uniformly weakly continuous.

c) If f is Henkin-continuous and tightens g, then also g is Henkin-continuous.

d) If f and $g :\subseteq Y \rightrightarrows Z$ are Henkin-continuous, then so is their composition $g \circ f :\subseteq X \rightrightarrows Z$.

e) A function $F :\subseteq \{\mathtt{0},\mathtt{1}\}^\omega \to \{\mathtt{0},\mathtt{1}\}^\omega$ is an (α,β)–realizer of f
iff F tightens $\beta^{-1} \circ f \circ \alpha$ iff $\beta \circ F \circ \alpha^{-1}$ tightens f.

f) If $\mathrm{range}(f) \subseteq \mathrm{dom}(g)$ holds and if both f and g map compact sets to compact sets,
then so does $g \circ f$.