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Algebraic, Topological, and Physical Aspects of Computing

SS 2012, Exercise Sheet #6

EXERCISE 6:

a) Prove that integration, i.e. the following mapping, is $\big([\rho\to\rho]\times\rho\times\rho,\rho\big)$–computable:

$$C(\mathbb{R})\times\mathbb{R}\times\mathbb{R} \;\ni\; (f,a,b) \;\mapsto\; \int_a^b f(t)\,dt \;\in\; \mathbb{R}$$

b) Generalize to indefinite and to higher dimensional integration.
How do you represent domains?

c) Prove that $\{\emptyset\}$ is $\psi_>^d\big|^{[0,1]^d}$–r.e. and
that the mapping $\big\{\{\vec{x}\}:\vec{x}\in[0,1]^d\big\}\ni\{\vec{x}\}\mapsto\vec{x}\in[0,1]^d$ is $(\psi_>^d,\rho^d)$–computable.

d) Derive from ρ and $[\rho^d\to\rho]$ representations γ and $[\gamma\to\gamma]$ for complex numbers and univariate continuous complex functions. Formalize and prove complex path integration computable.

e) Prove that the set $\mathbb{C}_c$ of computable complex numbers is algebraically closed.
How about the set $\mathbb{R}_c$ of computable reals?