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Algebraic, Topological, and Physical Aspects of Computing

SS 2012, Exercise Sheet #4

EXERCISE 4:

a) Suppose $f :\subseteq \mathbb{R} \to \mathbb{R}$ is (ρ, ρ)–computable and nondecreasing.
Conclude that f is $(\rho_<, \rho_<)$–computable and $(\rho_>, \rho_>)$–computable.

b) We know that $0 \le y \mapsto \sqrt{y}$ is (ρ, ρ)–computable. Now suppose closed $A \subseteq \mathbb{R}^d$ is $\psi^d_>$–computable.
Prove that $\mathrm{dom}(f) \ni \vec{x} \mapsto \sqrt{\|\vec{x}\|_2^2 - \mathrm{dist}(\vec{x}, A)^2}$ is $(\rho^d, \rho_>)$–computable.

c) Show that $\mathbb{R}$ is *computably* Archimedean in that the multivalued function $\mathbb{R} \ni x \rightrightarrows [x, \infty) \cap \mathbb{Z}$ is computable; but there exists no computable singlevalued $f : \mathbb{R} \to \mathbb{Z}$ with $\forall x : f(x) \ge x$.

d) Show that the following multivalued mapping is not computable for $d \ge 2$:

$$\mathsf{MLPO}_d : \{(x_1, \ldots, x_d) : x_i \in \mathbb{R}, \exists j : x_j = 0\} \ni (x_1, \ldots, x_d) \rightrightarrows \{j : 1 \le j \le d, x_j = 0\}$$

e) Show that $\mathsf{LSolve} :\subseteq \mathbb{R}^{d \times d} \setminus \mathrm{GL}(\mathbb{R}^d) \rightrightarrows \ker(A) \setminus \{\vec{0}\}$ is not computable for $d \ge 2$.

f) Linear independence $\{(\vec{x}_1, \ldots, \vec{x}_k) \in \mathbb{R}^{d \times k} \text{ linearly independent}\}$ is open, in fact $\rho^{d \times k}$–r.e.