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Algebraic, Topological, and Physical Aspects of Computing

SS 2012, Exercise Sheet #3

EXERCISE 3:

a) Every $(\rho_< \to \rho_<)$–computable $f : [0,1] \to \mathbb{R}$ is nondecreasing.
How about $(\rho_> \to \rho_>)$–computable f ?

b) $\mathbb{R}^2 \ni (x,y) \mapsto \max(x,y)$ is $(\rho_< \times \rho_< \to \rho_<)$–computable
and $(\rho_> \times \rho_> \to \rho_>)$–computable;
same for $(x,y) \mapsto \min(x,y)$.

c) For every computable real $p \geq 1$ and $d \in \mathbb{N}$,
the p-norm $\mathbb{R}^d \ni \vec{x} \mapsto \|\vec{x}\|_p = (\sum_j |x_j|^p)^{1/p}$ is (ρ^d, ρ)–computable a function.

d) Show that $\psi^d_>$–computability is independent of
the computable norm underlying the distance function.

e) Every nonempty $\psi^d_<$–computable set contains a computable point.
More uniformly, the multivalued mapping $\mathcal{A}^{(d)} \setminus \{\emptyset\} \ni A \rightrightarrows A$ is $(\psi^d_<, \rho^d)$–computable.

f) Is differentiation $C^1[0,1] \ni f \mapsto f' \in C[0,1]$ $\big([\rho \to \rho], [\rho \to \rho]\big)$–computable?