Algebraic, Topological, and Physical Aspects of Computing SS 2012, Exercise Sheet #1

EXERCISE 1:

- a) Let $(a_n)_n$ denote a computable sequence, $R := 1/\limsup_n \sqrt[n]{|a_n|}$ and 0 < r < R. Prove that $[-r, +r] \ni x \mapsto \sum_n a_n x^n$ is computable.
- b) Conclude that exp, \sin , \cos , $\ln(1 + x)$ are computable.
- c) Assert the restriction sgn $|_{\mathbb{R}\setminus\mathbb{Q}}$ to be computable.
- d) Prove that any computable $x \in \mathbb{R}$ is also binarily computable.
- e) Addition $[0,1] \times [0,1] \ni (x,y) \to x + y \in [0,2]$ is not $(\rho_b \times \rho_b \to \rho_b)$ -computable.
- f) Show that every $(\rho_n \rightarrow \rho_n)$ -computable $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous.
- g) Formalize real computation by approximation up to relative error; show that addition is not computable in this sense.

Would you prefer this lecture and/or exercises to be moved to a different date? Please indicate your preferences at doodle.de/nwaysuehnbcgi9p4