

Algebraic, Topological, and Physical Aspects of Computing

SS 2012, Exercise Sheet #1

EXERCISE 1:

- a) Let $(a_n)_n$ denote a computable sequence, $R := 1/\limsup_n \sqrt[n]{|a_n|}$ and $0 < r < R$.
Prove that $[-r, +r] \ni x \mapsto \sum_n a_n x^n$ is computable.
- b) Conclude that $\exp, \sin, \cos, \ln(1+x)$ are computable.
- c) Assert the restriction $\operatorname{sgn}|_{\mathbb{R} \setminus \mathbb{Q}}$ to be computable.
- d) Prove that any computable $x \in \mathbb{R}$ is also binarily computable.
- e) Addition $[0, 1] \times [0, 1] \ni (x, y) \rightarrow x + y \in [0, 2]$ is not $(\rho_b \times \rho_b \rightarrow \rho_b)$ -computable.
- f) Show that every $(\rho_n \rightarrow \rho_n)$ -computable $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous.
- g) Formalize real computation by approximation up to relative error;
show that addition is not computable in this sense.