Algorithmic Discrete Mathematics 6. Exercise Sheet

TECHNISCHE UNIVERSITÄT DARMSTADT

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Groupwork

Exercise G1 (Heap-Sort)
Use Heap-Sort with a min-heap to sort the array $(3,1,4,1,5,9,2)$.

Exercise G2 (Maximal flows)
Let $D=(V,E)$ be a directed graph with source s and sink t. The capacities on the edges should be natural numbers and the maximal flow x from s to t is given. Now we want to modify the capacity of one edge in the graph by

(a) increasing it by 1,

(b) decreasing it by 1.

Give an algorithm that calculates the maximal flow from s to t in that changed graph in $O(|V|+|E|)$.

Exercise G3 (Mengers Theorem)

(a) A directed graph $D=(V,E)$ is called *strongly k-connected*, if for every pair of vertices (u,v) and every set of edges $B\subseteq E$ with $|B|\leq k-1$, there exists a directed path from u to v in $D'=(V,E\setminus B)$. Prove the edge version of Mengers Theorem.
Mengers Theorem for edges: A directed graph $D=(V,E)$ is strongly k-connected, iff for every pair of vertices (s,t) there exist at least k directed paths from s to t which share no edges.

(b) A directed graph $D=(V,E)$ is called *k-vertex-connected*, if for every pair of vertices (u,v) and every set of vertices $W\subseteq V$ with $|W|\leq k-1$ there exists a directed path from u to v in $D-W$. Prove the vertex version of Mengers Theorem.
Mengers Theorem for vertices: A directed graph $D=(V,E)$ is k-vertex-connected, iff for every pair of vertices (s,t) there exist at least k directed paths from s to t sharing no vertices.

(c) Do Mengers Theorems also hold for undirected graphs?

Homework

Exercise H16 (10 points)
Look at the following variant of the quicksort algorithm.
Algorithm QuickSort(a,l,r)
Input: An array a of length n with $a[i]\in\mathbb{Z}$, lower and upper bounds l,r with $1\leq l\leq r\leq n$.
Output: The array a with $a[l]\leq a[l+1]\leq\ldots\leq a[r]$.

(1) Set $i=l-1$ and $j=r$.

(2) `While` $i<j$ `Do`

(3) `Do i = i + 1 While` $a[i]\leq a[r]$.

(4) `Do j = j - 1 While` $(a[j]\geq a[r]$ and $j\geq i)$.

(5) `If` $j>i$ `Then` Swap $a[j]$ and $a[i]$.

(6) `End While`

(7) Swap $a[i]$ and $a[r]$.

(8) `If` $l<i-1$ `Then` QuickSort $(a,l,i-1)$.

(9) If $i+1 < r$ Then QuickSort $(a, i+1, r)$.

(10) return a.

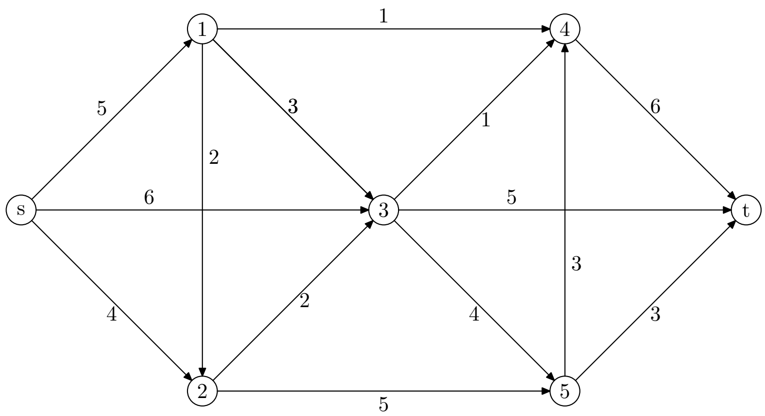
The iteration starts by using QuickSort(a,1,*length*(a)).

(a) Use Quicksort to sort the number array $(12, 3, 8, 13, 5, 2, 9, 4, 5, 3, 7)$.

(b) How does a number array have to look like, so that Quicksort has a runtime of $O(n^2)/O(n \log n)$. Describe it in general and give an example for both cases by using the first 10 natural numbers.

Exercise H17 (Finding maximal flows) (10 points)

Find the maximal flow from s to t in the following directed graph and prove its maximality.



Exercise H18 (Right or Wrong?) (10 points)

Let $D = (V, E)$ be a directed graph with integer capacities C_e for the edges $e \in E$. Let s be the source in this graph and t the sink. Which of the following statements are right? Which are wrong? Give a proof or a counterexample.

(a) If all capacities are even, then there exists a maximal flow f from s to t, such that $f(e)$ is even for all edges $e \in E$.

(b) If all capacities are odd, then there exists a maximal flow f from s to t, such that $f(e)$ is odd for all edges $e \in E$.

(c) If f is a maximal flow from s to t, then either $f(e) = 0$ or $f(e) = c_e$ holds for all edges $e \in E$.

(d) There exists a maximal flow from s to t, such that either $f(e) = 0$ or $f(e) = c_e$ holds for all edges $e \in E$.

(e) If all capacities on the edges are different, then the minimal cut is unique.

(f) If we multiply each capacity with the positive real number $\lambda \in \mathbb{R}_+$, then every minimal cut in there original graph is a minimal cut in the modified graph.

(g) If we add the positive number $\lambda \in \mathbb{R}_+$ to each capacity, then every minimal cut in the orginal graph is a minimal cut in the modified graph.

Der Matheball

Dieses Jahr findet der 21. Ball der Mathematiker am 7.7.2012 um 20:00 Uhr statt.
Vorverkauf: dienstags 11:40Uhr, donnerstags 9:50Uhr im Fachschaftsraum 347 (Mathebau).
VVK-Preis: 12Euro ermäßigt (Studenten, Jugendliche, Mitarbeiter), sonst 14Euro.
An der Abendkasse gibt es 2 Euro Aufschlag.
Je früher ihr die Karten kauft, desto eher könnt ihr euch eure guten Plätze in der Halle aussuchen.
Weitere Infos auf www.mathebau.de/matheball
Wir freuen uns auf euch!