Algorithmic Discrete Mathematics
4. Exercise Sheet

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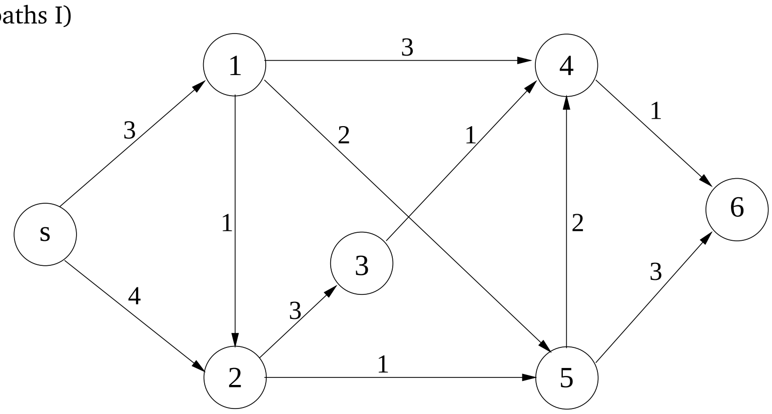
Groupwork

Exercise G1 (Shortest paths I)



(a) Calculate the shortest path from s to all other vertices by using the Dijstra algorithm. Determine the shortest path tree.

(b) Is the shortest path tree unique?

(c) Now change the weight of the edge $(3,4)$ to -2. Show that the Dijkstra algorithm does not work in this case.

Exercise G2 (Shortest paths II)
Show that the problem finding a shortest odd cycle in a simple digraph $G = (V, E)$ with nonnegative weights c_e on the edges can be solved by using a shortest path algorithm.

Exercise G3 (Crossing the river)
A man has to transport a wolf, a goat and a cabbage to the other side of a river. He has one boat to do this but it is so small that he can only take one of the three things with him each time. Is it possible to bring all three things to the other side of the river safely?

Notice that the wolf and the goat or goat and the cabbage must never be on the same side of the river without surveillance of the man. At least the wolf is no vegeterian and does not like to eat cabbage.

Homework

Exercise H10 (Algorithms) (10 points)
We are looking for an algorithm which gets an undirected, connected graph G given as an adjacency list and determines whether the graph is bipartite or not in runtime $O(|V| + |E|)$.

(a) Construct such an algorithm.

(b) Prove its correctness and analyse its runtime.

Exercise H11 (Graphs) (10 points)
Given the graph G in Figure 1.

(a) Is the graph G eulerian? Justify your answer.

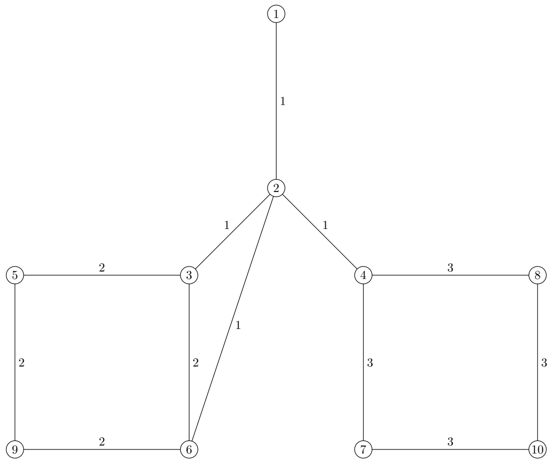


Figure 1: a graph

(b) Does the graph G contain a Hamiltonian cycle? This a cycle that contains each vertex exactly once. Justify your answer.

(c) Determine the adjacency matrix and adjacency list of the graph G.

Exercise H12 (Shortest paths) (10 points)

(a) Let $G = (V, E)$ be a directed graph.

i. Assume G has no negative cycles and $(s = i_0, i_1, \ldots, i_k = t)$ is a shortest simple path from s to t. Then every subpath from s to i_j with $j \in \{0, \ldots, k\}$ is a shortest simple path from s to i_j.

ii. Show that this statement is wrong for general graphs which may contain negative cycles.

(b) Given a directed graph $G = (V, E)$ we want to calculate the shortest path from the start to the destination. The problem is that start consists of n vertices and the destination consists of m vertices. So we are looking for the shortest path possibly starting at any s_i with $i \in \{1, \ldots, n\}$ and possibly going to any destination t_j with $j \in \{1, \ldots, m\}$. How can this be done efficiently? Justify your answer.

Hint: