# Algorithmic Discrete Mathematics 3. Exercise Sheet

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## Groupwork

**Exercise G1**
Let the algorithm CLIQUE be defined by:

**input** : A graph $G$ and a natural number $k$.

**output** : 'yes', if $G$ contains a clique of cardinality $k$. Otherwise 'no'.

Let the algorithm IDEPENDENT SET (IS) be defined by:

**input** : A graph $G$ and a natural number $k$.

**output** : 'yes', if $G$ contains an independent set consisting of $k$ vertices. Otherwise 'no'.

Show that CLIQUE $\leq_P$ IS.

**Exercise G2** (Bipartite graphs)
Prove that a graph $(V, E)$ is bipartite iff it contains no cycles of odd length.

**Exercise G3** (Eulerian graphs)
A path in a Graph $G = (V, E)$ is called *Eulerian path*, if it contains every edge $e \in E$ exactly once. An Eulerian path which is a cycle is called *Eulerian cycle*. A graph is called *eulerian* if it contains an Eulerian cycle.

(a) Which of the given graphs in **Figure 1** are Eulerian graphs?

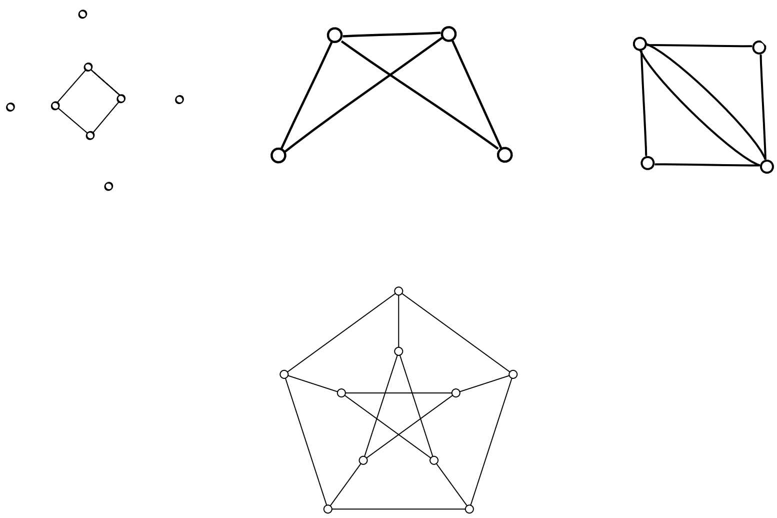

**Figure 1:** Eulerian graphs?

(b) Now let $G$ be a connected graph. Name necessary conditions for $G$ being eulerian.

(c) Are these conditions sufficient, too?

**Exercise G4** (Primes and the class $\mathcal{NP}$)
The class of problems whose complement is in $\mathcal{NP}$ is called co-$\mathcal{NP}$. For the following exercise assume that the coding length of a natural number $n$ is given by $\langle n \rangle = \lfloor \log_2 n \rfloor + 1$.[2]

(a) Show that the problem PRIMES, to determine if a given natural number is prime, is a co-$\mathcal{NP}$ problem.

(b) Why would it be much harder to show that this problem is also in the class $\mathcal{NP}$?

(c) Prove PRIMES $\in \mathcal{NP}$ under the assumption $\langle n \rangle = n$.

## Homework

**Exercise H7** (Trees) (10 points)
Let $G = (V, E)$ be a graph with $n \geq 2$ vertices. Proof that the following statements are equivalent:

(a) $G$ is a tree.

(b) $G$ is connected and contains $n - 1$ edges.

(c) $G$ contains $n - 1$ edges but no cycles.

(d) $G$ is minimally connected. That means for every edge $e \in E$ the graph $G \setminus \{e\} = (V, E \setminus \{e\})$ is not connected.

(e) $G$ contains no cycles and adding one edge generates exactly one cycle.

(f) For every two nodes $u, v \in V$ there is exactly one $[u, v]$-path in $G$.

**Exercise H8** (The complement graph) (10 points)
The complement graph $\overline{G}$ of $G = (V, E)$ is the graph were two vertices are adjacent iff they are not adjacent in $G$. So formally speaking we have $\overline{G} := (V, \binom{[n]}{2} \setminus E)$ with $\binom{[n]}{2} \setminus E := \{(i, j) \,|\, i \neq j \in \{1, \ldots, n\}(i, j) \notin E\}$.

Let $G$ be an undirected graph. Prove that $G$ or $\overline{G}$ is connected.

**Exercise H9** (10 points)
Prove SUBSETSUM $\leq_p$ PARTITION.

**problem** : SUBSETSUM

**input** : $a_1, \ldots, a_n, b \in \mathbb{N}$

**output** : $T \subseteq \{1, \ldots, n\}$ with $\sum_{k \in T} a_k = b$

**problem** : PARTITION

**input** : $a_1, \ldots, a_l \in \mathbb{N}$

**output** : $T \subseteq \{1, \ldots, l\}$ with $\sum_{k \in T} a_k = \sum_{k \notin T} a_k$

[2] This is the common binary representation of a natural number $n$. Hence you need $\lfloor \log_2 n \rfloor + 1$ digits.