The next weeks:

17.12.08 L   google
14.01.09 L   c/c++
19.01.09 E
21.01.09 L   chess(?), + talk
26.01.09 E
02.02.09 E
11.02.09 Exam

```
> restart; with(LinearAlgebra);
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LA_Main, LUDecomposition, LeastSquares, LinearSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]   **(1)**

We need:

1.) Discrete Probabilities

2.) Matrix operations: sums, matrix-matrix multiplication, matrix-vector multiplication

ad 1)

Let E be a set of discrete events.

First axiom: $0 \le P(e) \le 1$ for all $e \in E$. A Pobability is a number between 0 and 1

Second axiom: P(Ω) = 1. The probability that some event occurs is 1.
Third axiom: Let e(1),...,e(n) be pairwise disjoint events. Then

$$P(e(1) \cup \ldots \cup e(n)) = \sum_{i=1}^{n} P(e(i)).$$

Example:
Let the results of a mathematical coin-toss be heads or tails.
- event set is {heads, tails} #(dt: Wappen oder Zahl)
- as event space can be chosen
  ∑ = {{},{heads},{tails},{heads or tails}=Ω}
- For the probability measure is then fixed:
  P({}) = 0,
  P({heads}) = 1 - P({tails}),
  P(Ω) = 1.

ad 2)
a) Sum of matrices
The sum of two matrices

$A = (a_{ij})_{i=1..m, j=1..n}$ and $B = (b_{ij})_{i=1..m, j=1..n}$ is $C = (c_{ij}) = (a_{ij} + b_{ij})$

Example: $\begin{bmatrix} 44 & -31 \\ 92 & 67 \end{bmatrix} + \begin{bmatrix} 8 & 99 \\ 69 & 29 \end{bmatrix} = \begin{bmatrix} 52 & 68 \\ 161 & 96 \end{bmatrix}$

b) Multiplication of two matrices
The product of two matrices
A=
$(a_{ij})_{i=1..m, j=1..n}$ and $B = (b_{ij})_{i=1..n, j=1..o}$ is $C = (c_{ij}) = \left(\sum_{k=1}^{n} (a_{ik} + b_{kj}) \right)$

Example: $\begin{bmatrix} 44 & -31 \\ 92 & 67 \end{bmatrix} \cdot \begin{bmatrix} 8 & 99 \\ 69 & 29 \end{bmatrix} = \begin{bmatrix} (44 \cdot 8 - 31 \cdot 69) & (44 \cdot 99 - 31 \cdot 29) \\ (92 \cdot 8 + 67 \cdot 69) & (92 \cdot 99 + 67 \cdot 29) \end{bmatrix}$

c) is a special case of b)

>
>

The google problem:

given is

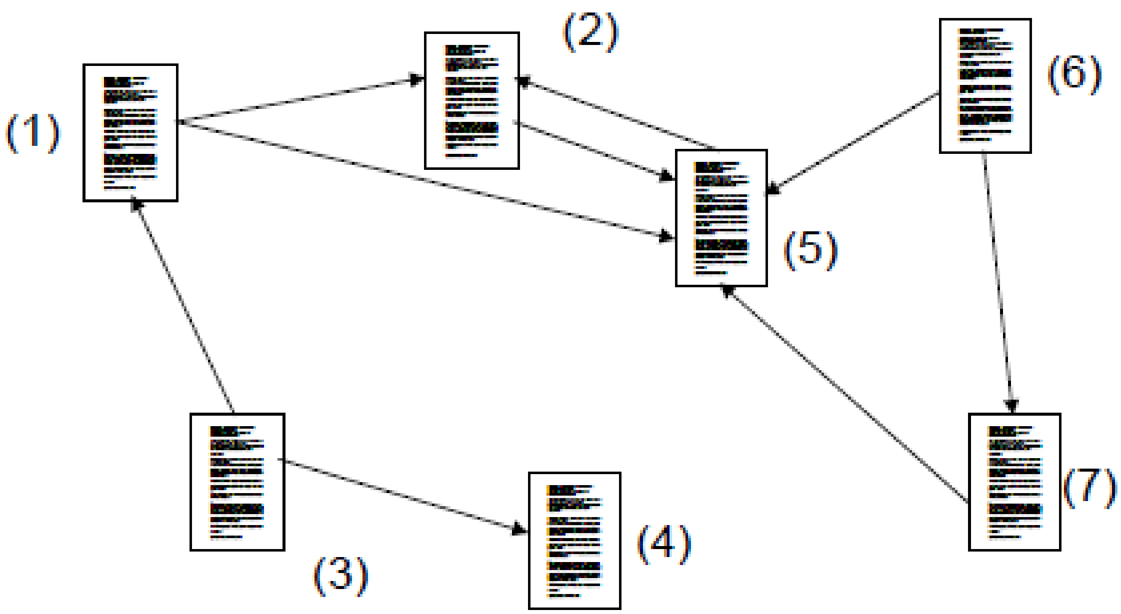
- a library with 25 billion documents
- no centralized organisation
- no librarians
- anyone can add documents

You are interested in information. You only know some keywords, and further complication:

Google claims **more then 25 billion indexed pages**. **95% of the text** in the Web is composed of **only some 1,000 words**. How can we distinguish the important pages from the unimportant ones?

Impossible?

The heart of the google software is the PageRank algorithm.



Let P be a web page.
We call Imp(P) the importance of P.

Let P_j have l_j many outgoing links to other pages.
If P_i is such a page, P_j will pass $1/l_j$ „importance" to P_i.

Let B_i be the set of pages linking to P_i. Then the importance relation between a page and its neighbours is as follows:

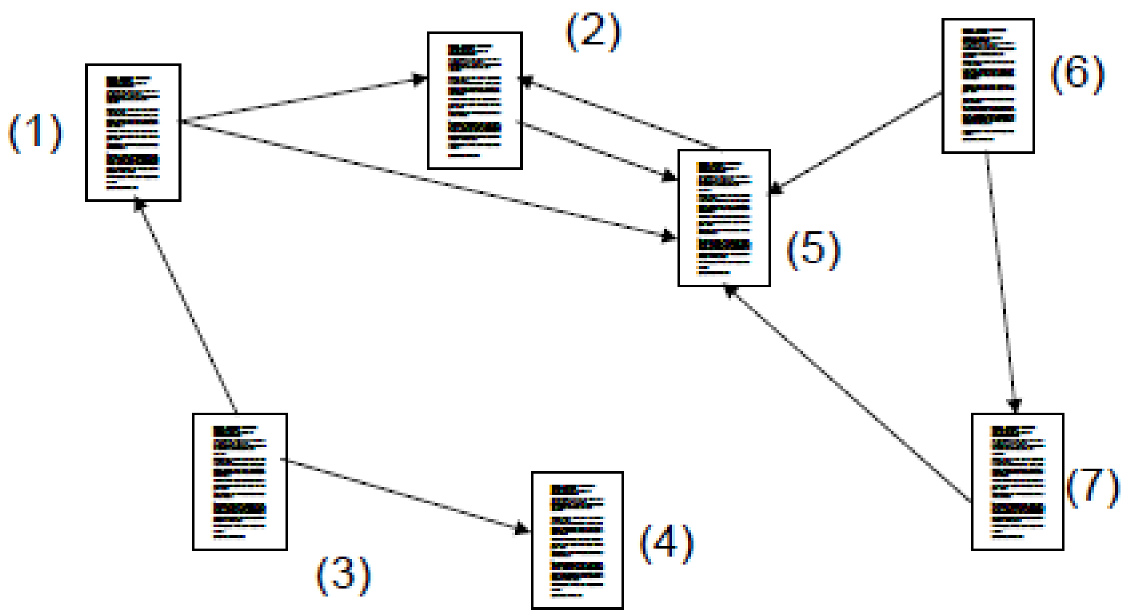
$$\text{Imp(Pi)} = \sum_{P_j \in B_j} \frac{Imp(P_j)}{l_j}$$

We already saw:

a correct PageRank assignment can be interpreted as the eigenvector Imp of a matrix H with eigenvalue 1, such that Imp = H*Imp:

Problem 1: Unfortunately, H contains so called dangling nodes, i.e. nodes without sucessors.

$$> H := \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix} : \#$$



Consequence: zero-columns => H not stochastic => possibly no stationary solution

> $(eigenvalues, eigenvectors) := LinearAlgebra[Eigenvectors](H);$

$$eigenvalues, eigenvectors := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

v is a vector of eigenvalues, e the matrix of all eigenvectors. The i-th eigenvalue corresponds to the i-th eigenvector.
-> good luck. This matrix has a solution.

Control:

> $Imp := Column(eigenvectors, [7]);$ *#remember: we are looking for an Imp with Imp = H*Imp*

$$Imp := \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

> $H.Imp, Imp$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

Now, let A be the matrix whose entries are all zero except for the columns of the dangling nodes, in which each entry is 1/n,
n being the number of nodes. Let S := H + A.

> $A := \begin{bmatrix} 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \end{bmatrix} : S := H + A :$

> $H, A, S;$

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{7} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{7} & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{7} & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & \frac{1}{7} & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} & 0 & \frac{1}{2} & 0 \end{bmatrix} \quad \textbf{(5)}$$

Now, for each column of S is valid that the entries of each column sum up to one. This guarantees the existence of a stationary vector. (*No proof here, but there exists a Theorem.*) S is called a "stochastic matrix".

--> New interpretation: there is a random surfer on the web. Which portion of time will he spend
in which node, if she decides her next jump concerning transition-probabilities as they are described in the matrix S?

Let us take a look at the solution with the help of matrix S:

> $(eigenvalues, eigenvectors) := LinearAlgebra[Eigenvectors](S);$

$$eigenvalues, eigenvectors := \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 1 \\ \frac{1}{14} + \frac{1}{14}\sqrt{15} \\ \frac{1}{14} - \frac{1}{14}\sqrt{15} \end{bmatrix}, \Bigg[\Bigg[\Bigg[0, -2, 0, 0, 0, \qquad \textbf{(6)}$$

$$-\frac{1}{2025} \frac{1}{\left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)^2 \left(-\frac{13}{14} + \frac{1}{14}\sqrt{15}\right)} \left(\left(2301 \left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)^3\right.\right.$$

$$\left.\left. - 166 \left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)^2 - \frac{247}{14} - \frac{65}{14}\sqrt{15}\right)\sqrt{15}\right),$$

$$\frac{1}{2025} \frac{1}{\left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 \left(-\frac{13}{14} - \frac{1}{14}\sqrt{15}\right)} \left(\left(2301 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^3\right.\right.$$

$$\left.\left. - 166 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 - \frac{247}{14} + \frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\Bigg],$$

$$\Bigg[-1, 1, 0, -1, 1, -\frac{1}{2025} \left(\left(-4166 \left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)^3 - 246 \left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)^2\right.\right.$$

$$\left.\left. + \frac{21}{2} - \frac{5}{2}\sqrt{15} - 8519 \left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)^4 + 32214 \left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)^5\right)\sqrt{15}\right)$$

$$\Bigg/ \left(\left(\frac{8}{7} + \frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14} + \frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14} + \frac{1}{14}\sqrt{15}\right)^2\right), \frac{1}{2025} \Bigg(\Bigg($$

$$-4166 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^3 - 246 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^2 + \frac{21}{2} + \frac{5}{2}\sqrt{15}$$

$$- 8519 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^4 + 32214 \left(\frac{1}{14} - \frac{1}{14}\sqrt{15}\right)^5\Bigg)\sqrt{15}\Bigg) \Bigg/ \Bigg(\Bigg(\frac{8}{7}$$

$$-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{2}\right)\Bigg],$$

$$\Bigg[0, 0, 0, 0, 0,$$

$$-\frac{2}{2025}\left(\left(2301\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^{3}-166\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^{2}-\frac{247}{14}\right.\right.$$

$$\left.\left.-\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\Bigg/\left(\left(\frac{8}{7}+\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}+\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)\right),$$

$$\frac{2}{2025}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{3}-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{2}-\frac{247}{14}\right.\right.$$
$$\left.\left.+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\Bigg/\left(\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)\right)$$

$$\Bigg],$$

$$\Bigg[0, 0, 0, 0, 0,$$

$$-\frac{14}{2025}\frac{1}{\left(\frac{8}{7}+\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}+\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^{3}\right.\right.$$

$$\left.\left.-166\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^{2}-\frac{247}{14}-\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right),$$

$$\frac{14}{2025}\frac{1}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\left(\left(2301\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{3}\right.\right.$$
$$\left.\left.-166\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^{2}-\frac{247}{14}+\frac{65}{14}\sqrt{15}\right)\sqrt{15}\right)\Bigg],$$

$$\Bigg[0, 1, 0, 1, 1, \frac{1}{15}\frac{177\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)^{2}-\frac{3}{14}+\frac{11}{14}\sqrt{15}}{\left(\frac{8}{7}+\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}+\frac{1}{14}\sqrt{15}\right)},$$

$$\frac{1}{15}\,\frac{177\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)^2-\frac{3}{14}-\frac{11}{14}\sqrt{15}}{\left(\frac{8}{7}-\frac{1}{7}\sqrt{15}\right)\left(-\frac{13}{14}-\frac{1}{14}\sqrt{15}\right)}\Bigg],$$

$$\left[0, 0, 0, 0, 0, \frac{2\left(\frac{1}{14}+\frac{1}{14}\sqrt{15}\right)}{\frac{8}{7}+\frac{1}{7}\sqrt{15}}, \frac{2\left(\frac{1}{14}-\frac{1}{14}\sqrt{15}\right)}{\frac{8}{7}-\frac{1}{7}\sqrt{15}}\right],$$

$$\left[1, 0, 0, 0, 0, 1, 1\right]\Big]$$

> $Imp := Column(eigenvectors, [5]), S.Imp;$

$$Imp := \begin{bmatrix}0\\1\\0\\0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\\1\\0\\0\end{bmatrix} \quad \textbf{(7)}$$

If Imp is a solution to our problem, then also 1/2*Imp is a solution: H * (1/2 * Imp) = 1/2 * Imp

> $\frac{1}{2}.Imp;$

$$\begin{bmatrix}0\\\frac{1}{2}\\0\\0\\\frac{1}{2}\\0\\0\end{bmatrix} \quad \textbf{(8)}$$

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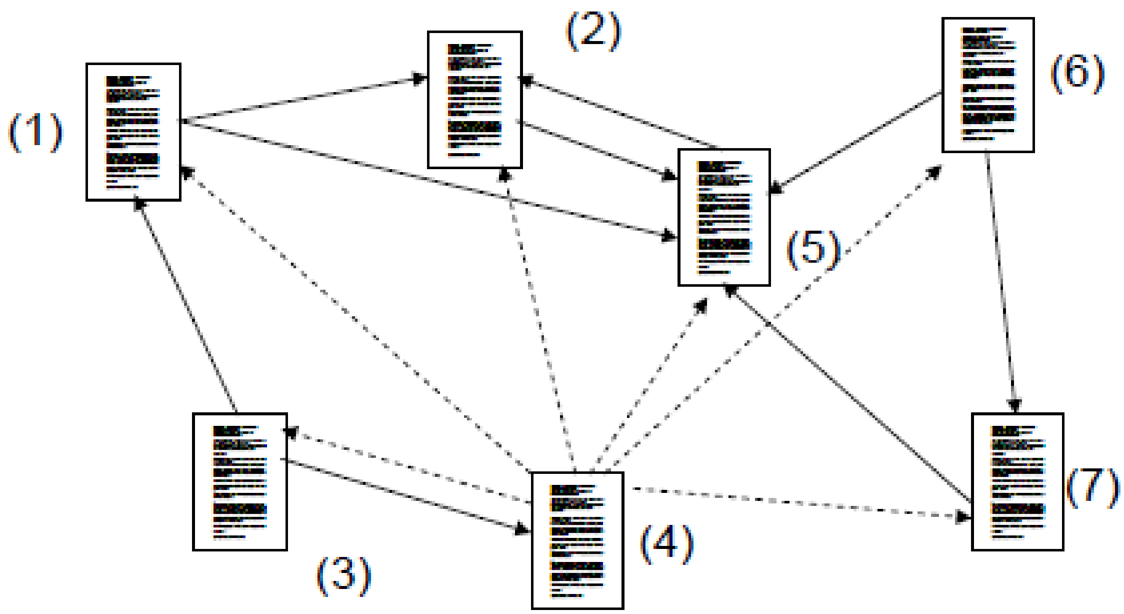

Unfortunately, there is Problem 2:

The nodes (2) and (5) are importance sinks.
-> In the graph, you see that the random walker is trapped
-> The graph is sayed to be "not strongly connected". It does not exists a path from any node to any other node.

-> The matrix is not "irreducible", i.e. S can be written in block form: S = $\begin{bmatrix} * & 0 \\ * & * \end{bmatrix}$.

Strongly connected graphs produce irreducible matrices.
(*No proof here, but there exists a Theorem.*)

$$> E := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix};$$

(9)

$$E := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \textbf{(9)}$$

> $G := \frac{85}{100} \cdot S + \left(1 - \frac{85}{100}\right) \cdot \frac{1}{7} \cdot E;$ # $\frac{1}{7}$ *because we have* 7 *nodes*

$$G := \begin{bmatrix} \frac{3}{140} & \frac{3}{140} & \frac{25}{56} & \frac{1}{7} & \frac{3}{140} & \frac{3}{140} & \frac{3}{140} \\ \frac{25}{56} & \frac{3}{140} & \frac{3}{140} & \frac{1}{7} & \frac{61}{70} & \frac{3}{140} & \frac{3}{140} \\ \frac{3}{140} & \frac{3}{140} & \frac{3}{140} & \frac{1}{7} & \frac{3}{140} & \frac{3}{140} & \frac{3}{140} \\ \frac{3}{140} & \frac{3}{140} & \frac{25}{56} & \frac{1}{7} & \frac{3}{140} & \frac{3}{140} & \frac{3}{140} \\ \frac{25}{56} & \frac{61}{70} & \frac{3}{140} & \frac{1}{7} & \frac{3}{140} & \frac{25}{56} & \frac{61}{70} \\ \frac{3}{140} & \frac{3}{140} & \frac{3}{140} & \frac{1}{7} & \frac{3}{140} & \frac{3}{140} & \frac{3}{140} \\ \frac{3}{140} & \frac{3}{140} & \frac{3}{140} & \frac{1}{7} & \frac{3}{140} & \frac{25}{56} & \frac{3}{140} \end{bmatrix} \quad \textbf{(10)}$$

> (*eigenvalues*, *eigenvectors*) := *LinearAlgebra*[*Eigenvectors*](*G*) :

> *eigenvalues*;

$$\begin{bmatrix} 1 \\ \frac{17}{280} + \frac{17}{280}\sqrt{15} \\ \frac{17}{280} - \frac{17}{280}\sqrt{15} \\ 0 \\ 0 \\ 0 \\ -\frac{17}{20} \end{bmatrix} \quad \textbf{(11)}$$

> *Imp* := *Column*(*eigenvectors*, [1]), *G.Imp*;

$$Imp := \begin{bmatrix} 1 \\ \frac{139559}{12654} \\ \frac{40}{57} \\ 1 \\ \frac{147413}{12654} \\ \frac{40}{57} \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ \frac{139559}{12654} \\ \frac{40}{57} \\ 1 \\ \frac{147413}{12654} \\ \frac{40}{57} \\ 1 \end{bmatrix} \qquad \textbf{(12)}$$

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The next question is, how we can compute the solution as fast as possible. The idea of the so called power method is to use the fact that under certain circumstances the sequence Imp^0 = a and Imp^(k+1) = H*Imp^k
converges to the correct solution.
It will do so, if the matrix G is irreducible and stochastic.(*There is a Theorem, no proof here*)

> $Start := \left\langle \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7} \right\rangle;$

$$Start := \begin{bmatrix} \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \\ \frac{1}{7} \end{bmatrix} \qquad \textbf{(13)}$$

> *G.Start*;

$$\begin{bmatrix} \frac{39}{392} \\ \frac{433}{1960} \\ \frac{19}{490} \\ \frac{39}{392} \\ \frac{79}{196} \\ \frac{19}{490} \\ \frac{39}{392} \end{bmatrix} \quad \textbf{(14)}$$

> $G^2.Start;$

$$\begin{bmatrix} \frac{13717}{274400} \\ \frac{45923}{109760} \\ \frac{1839}{54880} \\ \frac{13717}{274400} \\ \frac{200103}{548800} \\ \frac{1839}{54880} \\ \frac{13717}{274400} \end{bmatrix} \quad \textbf{(15)}$$

> $seq\left(evalf\left(G^k.Start\right), k = 8..11\right);$

$$\begin{bmatrix} 0.03693512985 \\ 0.4239101952 \\ 0.02591652167 \\ 0.03693512985 \\ 0.4134513719 \\ 0.02591652167 \\ 0.03693512985 \end{bmatrix}, \begin{bmatrix} 0.03692807319 \\ 0.3930446478 \\ 0.02591355148 \\ 0.03692807319 \\ 0.4443440297 \\ 0.02591355148 \\ 0.03692807319 \end{bmatrix}, \begin{bmatrix} 0.03692595398 \\ 0.4192995509 \\ 0.02591269460 \\ 0.03692595398 \\ 0.4180971979 \\ 0.02591269460 \\ 0.03692595398 \end{bmatrix}, \begin{bmatrix} 0.03692533247 \\ 0.3969885859 \\ 0.02591243727 \\ 0.03692533247 \\ 0.4404105421 \\ 0.02591243727 \\ 0.03692533247 \end{bmatrix} \quad \textbf{(16)}$$

> $seq(evalf(G^k.Start), k = 100..103);$

$$\begin{bmatrix} 0.03692507018 \\ 0.4072408675 \\ 0.02591232995 \\ 0.03692507018 \\ 0.4301592621 \\ 0.02591232995 \\ 0.03692507018 \end{bmatrix}, \begin{bmatrix} 0.03692507018 \\ 0.4072408576 \\ 0.02591232995 \\ 0.03692507018 \\ 0.4301592720 \\ 0.02591232995 \\ 0.03692507018 \end{bmatrix}, \begin{bmatrix} 0.03692507018 \\ 0.4072408660 \\ 0.02591232995 \\ 0.03692507018 \\ 0.4301592636 \\ 0.02591232995 \\ 0.03692507018 \end{bmatrix}, \begin{bmatrix} 0.03692507018 \\ 0.4072408588 \\ 0.02591232995 \\ 0.03692507018 \\ 0.4301592707 \\ 0.02591232995 \\ 0.03692507018 \end{bmatrix} \quad (17)$$

>

Last but not least: what about matrices with 25 billion rows and columns?

→ Remember : S = H + A

therefore : $G = 0.85 \cdot H + 0.85 \cdot A + \frac{(1 - 0.85)}{n} \cdot E$

therefore : $G \cdot Imp^k = 0.85 \cdot H \cdot Imp^k + 0.85 \cdot A \cdot Imp^k + \frac{(1 - 0.85)}{n} \cdot E \cdot Imp^k$

now : *most entries of H are zero. The rows of A are all the same*, **and** *the rows of E are all the same.*
therefore : *In practice only about* 300 *billion operations.*

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