

## Points, Vectors, and Matrices

```
> with(LinearAlgebra); with(plots);
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm,
BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column,
ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation,
CrossProduct, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix,
Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors,
Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations,
GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix,
GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm,
HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite,
IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LA_Main,
LUDecomposition, LeastSquares, LinearSolve, Map, Map2, MatrixAdd, MatrixExponential,
MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower,
MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular,
Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent,
Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank,
RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation,
RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues,
SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix,
ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix,
VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply,
ZeroMatrix, ZeroVector, Zip]
```

```
[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, (1)
conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot,
display, fieldplot, fieldplot3d, gradplot, gradplot3d, graphplot3d, implicitplot,
implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot,
listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple,
odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d,
polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions,
setoptions3d, spacecurve, sparsematrixplot, surldata, textplot, textplot3d, tubeplot]
```

Let us inspect (column) vectors.

```
> p := <0, 1>; r := <1, 2>;
```

$$p := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$r := \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (2)$$

```

> p[1];
0
(3)

> r[2];
2
(4)

> l := p + λ · r;
l := 
$$\begin{bmatrix} \lambda \\ 1+2\lambda \end{bmatrix}$$

(5)

Now, we want to compute the shortest distance from point q := <2,1> to the line.
> q := <2, 1>;
q := 
$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

(6)

> lineplot := plot([l[1], l[2], λ=-2..2]);
lineplot := PLOT(...)
(7)

> f := λ → p + λ · r;
f := λ → p + λ r
(8)

> s := seq([l[1], l[2]], λ=-2..2);
s := [-2, -3], [-1, -1], [0, 1], [1, 3], [2, 5]
(9)

> t := seq(f(x/10)[1], f(x/10)[2], x=-20..20):
> pointline := pointplot([s]);
pointline := PLOT(...)
(10)

> Qplot := pointplot(q);
Qplot := PLOT(...)
(11)

> a1 := arrow([0, 0], p, width=[0.075, relative=false], head_length=[0.4, relative=false],
color=green);
a1 := PLOT(...)
(12)

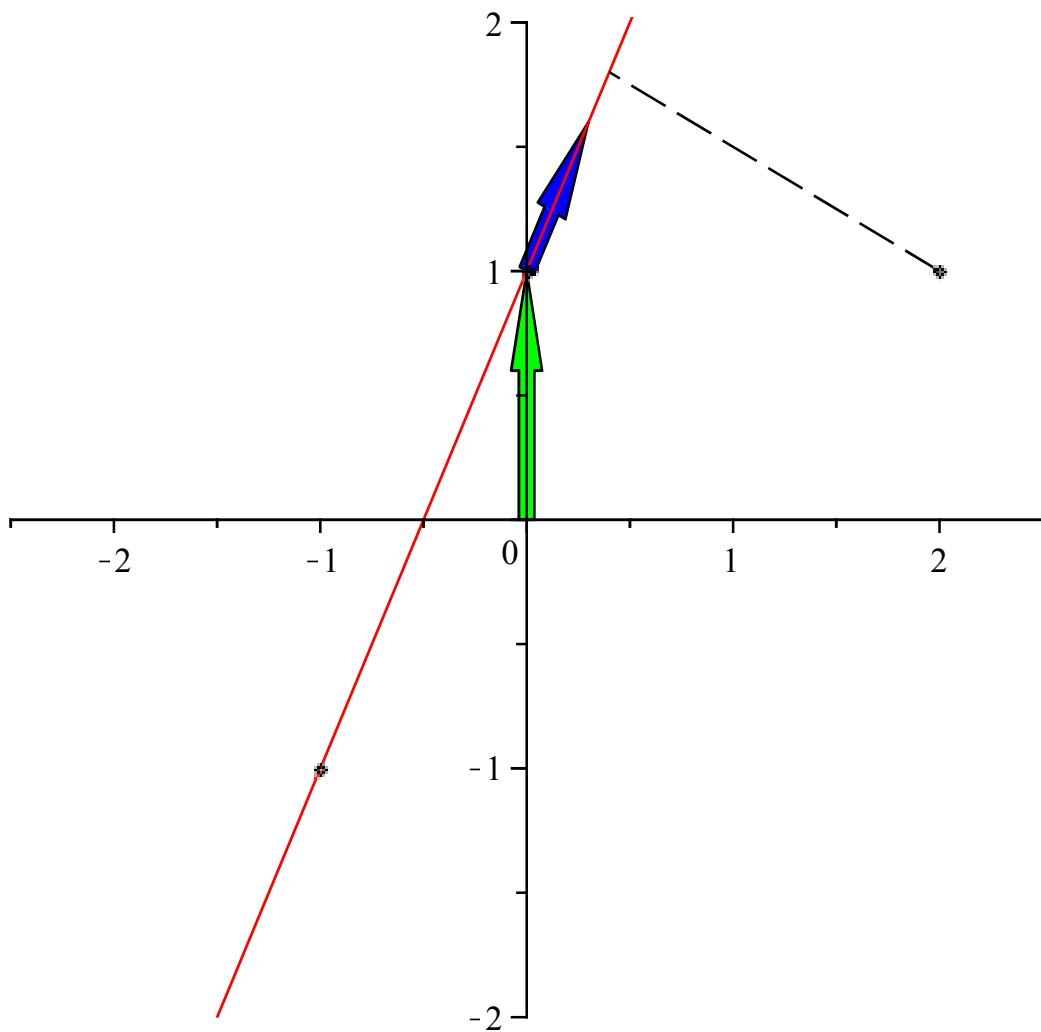
> a2 := arrow(p, 0.3 · r, width=[0.075, relative=false], head_length=[0.4, relative=false],
color=blue);
a2 := PLOT(...)
(13)

> HitPoint := subs(λ =  $\frac{\text{DotProduct}(q - p, r)}{\text{DotProduct}(r, r)}$ , l);
HitPoint := 
$$\begin{bmatrix} \frac{2}{5} \\ \frac{9}{5} \end{bmatrix}$$

(14)

> display([a1, a2, lineplot, pointline, Qplot, pointplot([q, HitPoint], connect=true, thickness=1, linestyle=dash)], view=[-2.5 .. 2.5, -2 .. 2]);

```



```

> myDotProduct := proc(u, v, k)
  local j, res, lu, lv;
  res := 0;
  for j from 1 to k do
    res := res + u[j]·v[j];
  end do;
  #return res;
  lu := convert(u,'list');
  lv := convert(v,'list');
  return  $\sum_{i=1}^k lu[i] \cdot lv[i]$ ;
end proc;
>  $\frac{\text{DotProduct}(q-p, r)}{\text{DotProduct}(r, r)}$ ;
 $\frac{2}{5}$ 
>  $\frac{\text{myDotProduct}(q-p, r, 2)}{\text{myDotProduct}(r, r, 2)}$ ;

```

(15)

(16)

$$\frac{2}{5}$$

>