## Maple

## Properties

- Software package
- implemented in the programing language C
- available for many Operating Systems, e.g. Windows, Unix, Linux
- desined for numerical and symbolic expressions
- includes untilities for algebra, calculus, discrete mathematics, graphics, ...


## History

- 1980: first development at the University of Waterloo, Canada
- 1988: Waterloo Maple Software was founded in order to sell and improve the software
- currently: version 12


## Getting started

- login to one of the machines in the pool in the Piloty building
- open a shell / a terminal
- type: xmaple (or maple, if you would like to work without windows; e.g. remote from home)


Menu bar at the top:

- allows you to save or load and edit your maple session
e.g. clicking on the File menu and selecting Save allows to save the current worksheet
- below the menu bar, there is a collection of shortcut-buttons

Maple Help

- help menu, "Maple Help"
- ?command; e.g. ?solve, if you know the keyword in advance
- the help-window has two panels: the Help Navigator on the left and the help itself on the right
- each help page contains some examples; copying an example and pasting it into the worksheet is possible


## Basic Conventions

## Entering a command, example

$$
\left[\begin{array}{ll}
> & \text { restart } ; \\
\gg 3+4
\end{array}\right.
$$

$$
\begin{equation*}
7 \tag{1}
\end{equation*}
$$

## Arithmetic operators

| Addition | + | $3+4$ |
| :--- | :--- | :--- |
| Substractio <br> $n$ | - | $x-y$ |
| Multiplicatil <br> on | $*$ | $2^{*} \mathrm{x}$ |
| Division | $/$ | $\mathrm{x} / \mathrm{y}$ |
| Exponentiai <br> ion | $\wedge$ | $3^{\wedge} 4$ |
| Factorial | $!$ | $3!$ |

The precedence order follows the mathematical conventions:
$\begin{array}{ll}{\left[\begin{array}{ll}> & (10-1) \cdot 2 \\ & \\ > & 10-1 \cdot 2\end{array}\right.} & 18 \\ {[>} & 8\end{array}$
Special commands to access previous results
\% latest one
\%\% secoond most recent
\%\%\% third most recent

```
> \#this is a comment
\(>2 \cdot 4\); \# most recent result becomes 8
                                    8
\(>\% \cdot 12.4\); \# this computes \(8 \cdot 12.4\). 99.2 becomes most recent result 99.2
\(>\% \%-\%\); \# computes 8-99.2
\[
-91.2
\]
Defining Expressions with ":="
- expression: combination of numbers, variables and operators
- Syntax is name:=expression
- maybe most used concept in Maple
Example
```

```
\(>f:=(1-x)^{2} \cdot x^{3}\);
```

$>f:=(1-x)^{2} \cdot x^{3}$;
$f:=(1-x)^{2} x^{3}$

```
    \(f:=(1-x)^{2} x^{3}\)
```

If you make a mistake, you can go back with the cursor, change the command-line and re-execute the line.

## Basic Data Structures

- fundamental data structures: expression sequences, lists, sets. (e.g. used as parameters in maple commands)

Sequences, implicitely or with command $\operatorname{seq}(\mathrm{f}(\mathrm{i}), \mathrm{i}=\mathrm{m} . . \mathrm{n})$
$>3,5, x, 4$;

$$
\begin{gather*}
3,5, x, 4  \tag{8}\\
s:=3,5, x, 4  \tag{9}\\
3,5, x, 4  \tag{10}\\
t:=4,9,16,25 \tag{11}
\end{gather*}
$$

A list

- is an expression sequence enclosed in square brackets
- preserves order and repetition of elements

A set

- is an expression sequence enclosed in curly brackets
- does not preserve order an does not contain the same element several times

$$
\begin{array}{ll}
\gg \text { list } 1:=[5,4,3,5,4,3] ; & \text { list } 1:=[5,4,3,5,4,3] \\
=>\text { list } 2:=[3,4,5] ; & \text { list } 2:=[3,4,5] \\
\hline>\text { set } 1:=\{5,4,3,5,4,3\} ; & \text { set }:=\{3,4,5\} \\
\hline>\text { set } 2:=\{4,5,3\} ; & \text { set } 2:=\{3,4,5\}
\end{array}
$$

## Numerical Computation

## Fraction numbers and floating point numbers

- fractions are not reduced to floating point approximations
- exact computations with fractions
- with evalf, the fraction can be converted to a floatring point number with Digits many digits.
$\left[>x:=\frac{9}{8}+\frac{6}{5} ;\right.$

$$
\begin{equation*}
x:=\frac{93}{40} \tag{16}
\end{equation*}
$$

$>\operatorname{evalf}(\%)$;
2.325000000
$>\operatorname{evalf}(x)$;
2.325000000
$>$ Digits $:=20 ;$

$$
\begin{equation*}
\text { Digits }:=20 \tag{19}
\end{equation*}
$$

$>\operatorname{evalf}(x)$;
2.3250000000000000000
$>\frac{9}{8.0}+\frac{6}{5} ; \#$ a floating number in the expression leads to implicit evalf
2.3250000000000000000
[ $>$
Integer numbers

- arbitrary large integers (as far as there is enough memory)

```
    121!;
80942985252734437396816228454493508299708230630970160704577623362849766042664\
    05217133917739979101827382870741850789049568566634393183827450477162148411\
    476507217602230720921600000000000000000000000000000
[>
```


## Complex Numbers

- a complex number $z$ is of the form $a+b i$, with $i^{2}=-1$ and $a, b \in \mathbb{R} . a=\operatorname{Re}(z)$ is the realpart of $z$ and $b=\operatorname{Im}(z)$
is the imaginary part of $z$
- two complex numbers are equal if and only if their real parts and their imaginary parts are equal
- Complex numbers are added, subtracted, multiplied, and divided by formally applying the associative,
commutative and distributive laws of algebra, together with the equation i $2=-1$.
Addition $:(\mathrm{a}+\mathrm{bi})+(\mathrm{c}+\mathrm{di})=(\mathrm{a}+\mathrm{c})+(\mathrm{c}+\mathrm{d}) \mathrm{i}$
Substraction : $(\mathrm{a}+\mathrm{bi})-(\mathrm{c}+\mathrm{di})=(\mathrm{a}-\mathrm{c})+(\mathrm{c}-\mathrm{d}) \mathrm{i}$
Multiplication: $(a+b i) \cdot(c+d i)=(a c-b d)+(b c+a d) i$
Division $: \frac{a+b i}{c+d i}=\frac{a c+b d}{c^{2}+d^{2}}+\frac{b c-a d}{c^{2}+d^{2}} i$, with c ord not equal to 0
- with the given definitions of addition, substraction, multiplication, division, and the additive identity (zero-element) $0+0$ i, the multiplicative identity (one-element) $1+0 \mathrm{i}$, the addidive inverse of a number a $+\mathrm{bi}:-\mathrm{a}-\mathrm{bi}$, and
the multiplicative inverse of $\mathrm{a}+\mathrm{bi}: \frac{a}{a^{2}+b^{2}}+\frac{-b}{a^{2}+b^{2}}$,
the complex numbers $\mathbb{C}$ are afield (dt: Körper)

$$
\left[\begin{array}{l}
>\frac{(3+3 \cdot I)}{(2+6 \cdot I)} ; \\
>\left(\frac{3}{3^{2}+5^{2}}+\frac{(-5)}{3^{2}+5^{2}} \cdot I\right) \cdot(3+5 \cdot I) ;
\end{array}\right.
$$

## Symbolic Computations

$$
\begin{align*}
& {\left[\begin{array}{l}
>\left(\frac{a}{a^{2}+b^{2}}+\frac{-b}{a^{2}+b^{2}} \cdot I\right) \cdot(a+b \cdot I) ; \\
\left(\frac{a}{a^{2}+b^{2}}-\frac{\mathrm{I} b}{a^{2}+b^{2}}\right)(a+\mathrm{I} b) \\
{[>} \\
\gg \text { simplify }(\%) ; \\
{[>\text { ?assume }}
\end{array}\right.} \\
& \hline> \tag{25}
\end{align*}
$$

The following expression leads to a surprising answer. Why? Because somewhere above, we already defined x . Thus: be careful and alert!

$$
\left\lceil>\operatorname{simplify}\left(\sin (x)^{2} \cdot x^{4}+\cos (x)^{2} \cdot x^{4}\right) ;\right.
$$

$$
\begin{equation*}
\frac{74805201}{2560000} \tag{27}
\end{equation*}
$$

$\left[>\operatorname{simplify}\left(\sin (y)^{2} \cdot y^{4}+\cos (y)^{2} \cdot y^{4}\right) ;\right.$

$$
\begin{equation*}
y^{4} \tag{28}
\end{equation*}
$$

$$
\begin{aligned}
& >\text { restart } \\
& >\operatorname{simplify}\left(\sin (x)^{2} \cdot x^{4}+\cos (x)^{2} \cdot x^{4}\right)
\end{aligned}
$$

$$
\begin{equation*}
x^{4} \tag{29}
\end{equation*}
$$

